Lecture 2: Signals + Systems F2016 (1)

What is a signal? : a function containing information about some phenomenon

e.g. functions of time
- acoustic (music, speech)
- sunlight
- EM (radio, cell phone)
- # tweets
- stock index
- ocean levels

↑

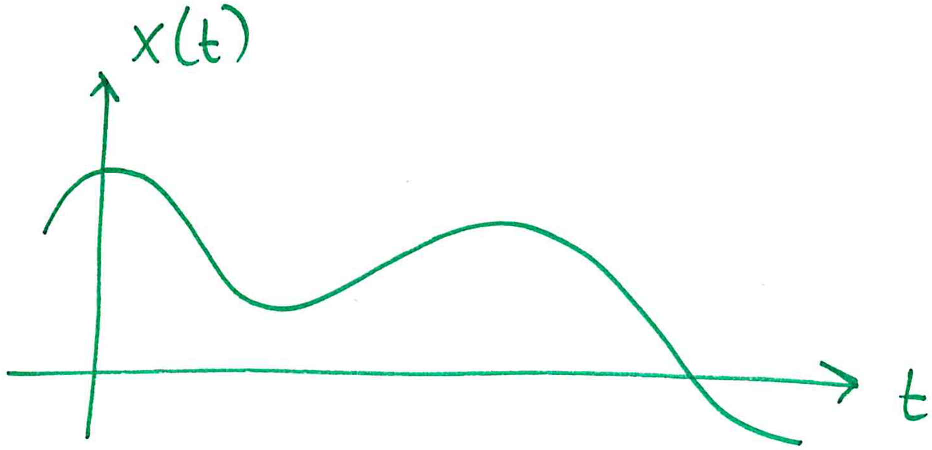
eg fuctions of space
- light (image / video)
- temperature
- text
- traffic
- epidemics

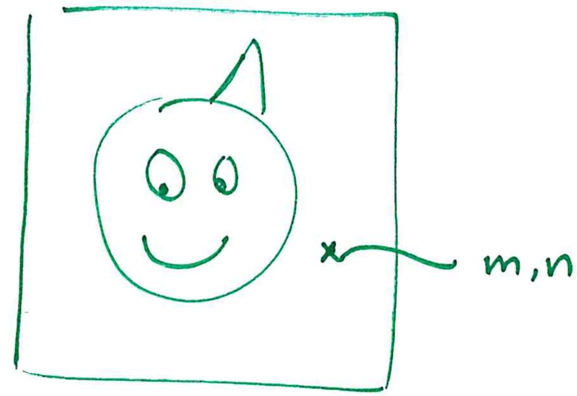
other
- netflix movie preferences
- Trump support
- health

What is system? anything that receives one signal and outputs another signal

System	Input	Output
Radio	EM wave	acoustic
Phone	Speech	EM wave
~~Xray~~ Body	X-ray	X-ray
Camera	Light	image

Signal = function $\Rightarrow$ we can represent it mathematically:





$x(m,n)$

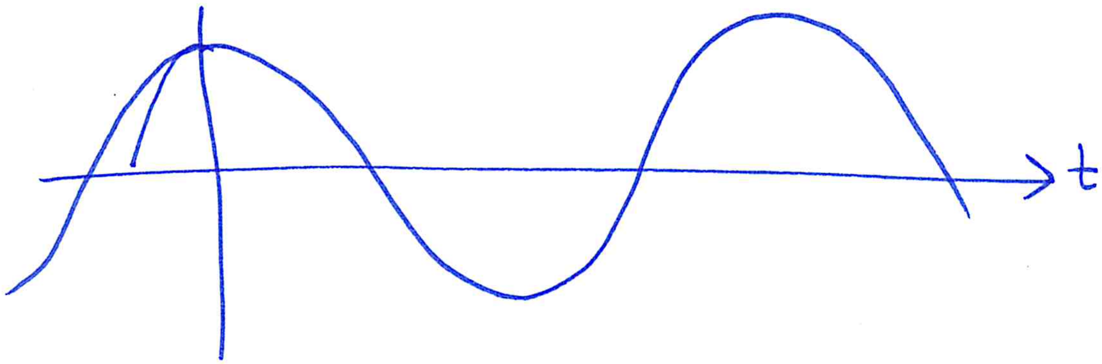
Systems transform signals $\Rightarrow$ represent a system as a mathematical operator

e.g. output $y(t) = (x(t))^2$

e.g $y(t) = 2x(t)$

This course, we will focus on sinusoids.

$x(t) = \cos(t)$



Why?

1. associated w/ physical models of real systems (tuning, crystal oscillations)
2. essential to models of communication signals (radio, music)
3. universal - any signal can be represented using sinusoids
4. systems can be understood through their effect on sinusions.

Sinusoid:

$$x(t) = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

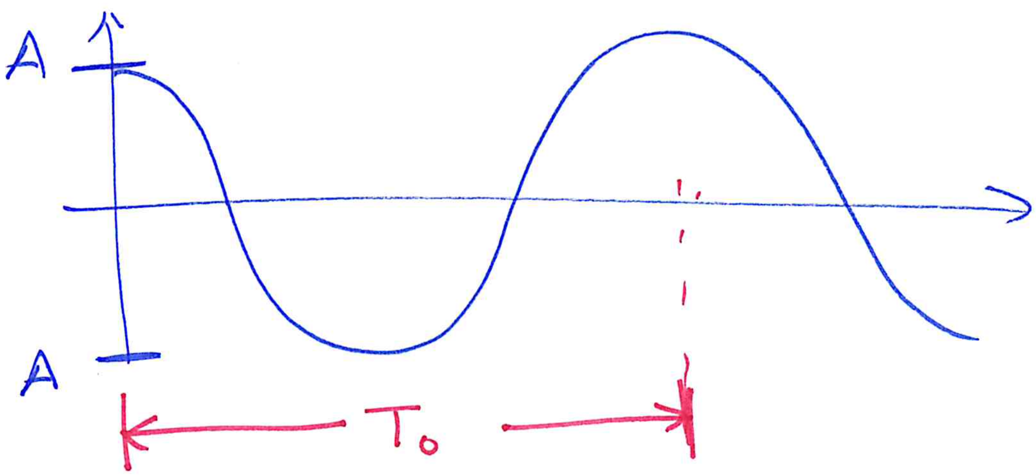
A – amplitude

ϕ – phase

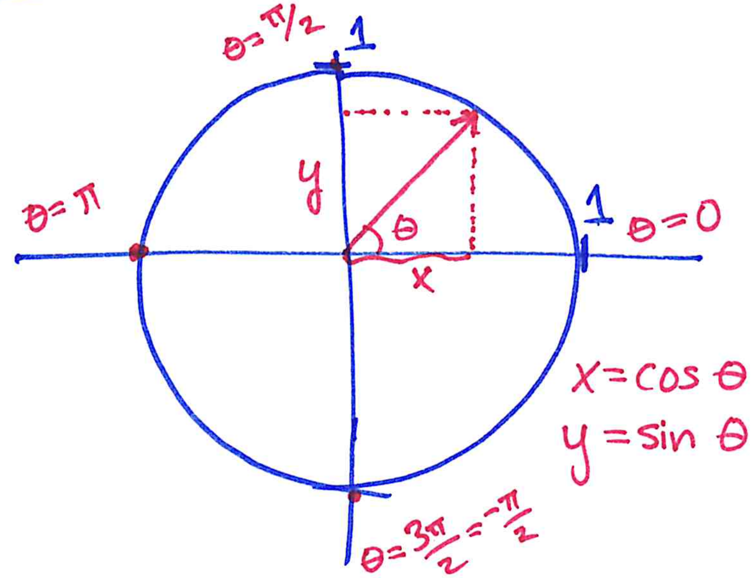
$\omega_0 = 2\pi f_0$ = frequency

$T_0 = 1/f_0 = 2\pi/\omega_0$ see

= length of 1 cycle



unit circle



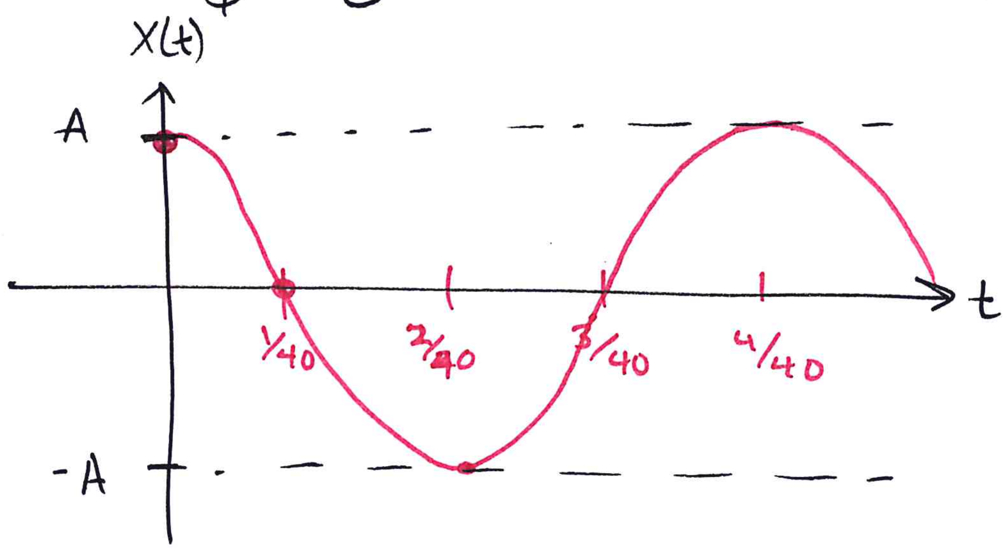
Ex:

$$x(t) = \underline{5} \cos(2\pi \cdot 10\, t)$$

$A = 5$

$f_0 = 10$ Hz $(\text{Hz} = 1/\text{sec}) \Rightarrow \omega_0 = 20\pi$, $T_0 = 1/10 = .1$ s

$\phi = 0$



@ $t=0$, $\theta = 2\pi \cdot 10 \cdot 0 = 0$

$\cos(\theta=0) = 1 \Rightarrow x(t) = 5$

@ $t = 0.025 = 1/40$ s.

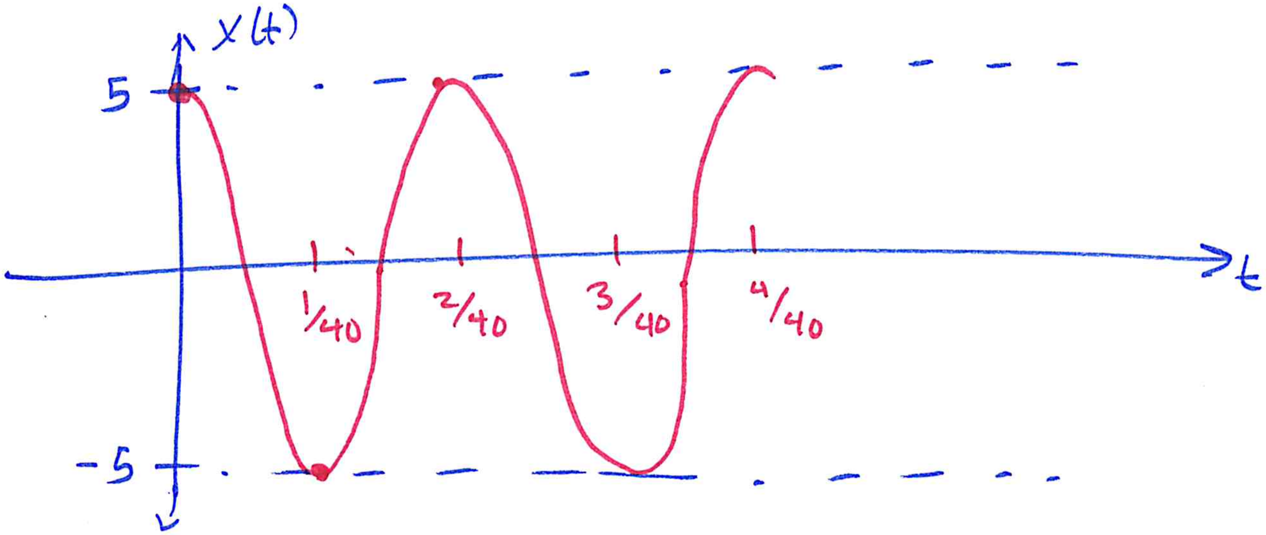
$\theta = 2\pi \cdot 10 \cdot 1/40 = \pi/2 \Rightarrow x(t) = 0$

@ $t = 0.05 = 1/20$ s.

$\theta = 2\pi \cdot 10/20 = \pi \Rightarrow x(t) = -5$

Ex: $x(t) = 5\cos(2\pi \cdot 20t)$

$f_0 = 20\ Hz \Rightarrow \omega_0 = 40\pi, \quad T_0 = \frac{1}{20} = .05\ s$



@ $t = 0, \ \theta = 0 \qquad x(t) = 5$

@ $t = .025 = 1/40$

$\theta = 2\pi \cdot 20/40 = \pi$

$x(t) = 5 \cdot \cos(\pi) = -5$

@ $t = .05 = 1/20$

$\theta = 2\pi \cdot 20/20 = 2\pi$

$x(t) = 5\cos(2\pi) = 5$

Sampling = reduction of a ~~continuo~~ continuous signal to a discrete signal.

Let T_s = "sampling period" = amount of time between samples / measurements of signal.

n^{th} sample at time $t = n \cdot T_s$

$$x[n] = x(\underbrace{n \cdot T_s}_{t})$$