Lecture 4

Euler's formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

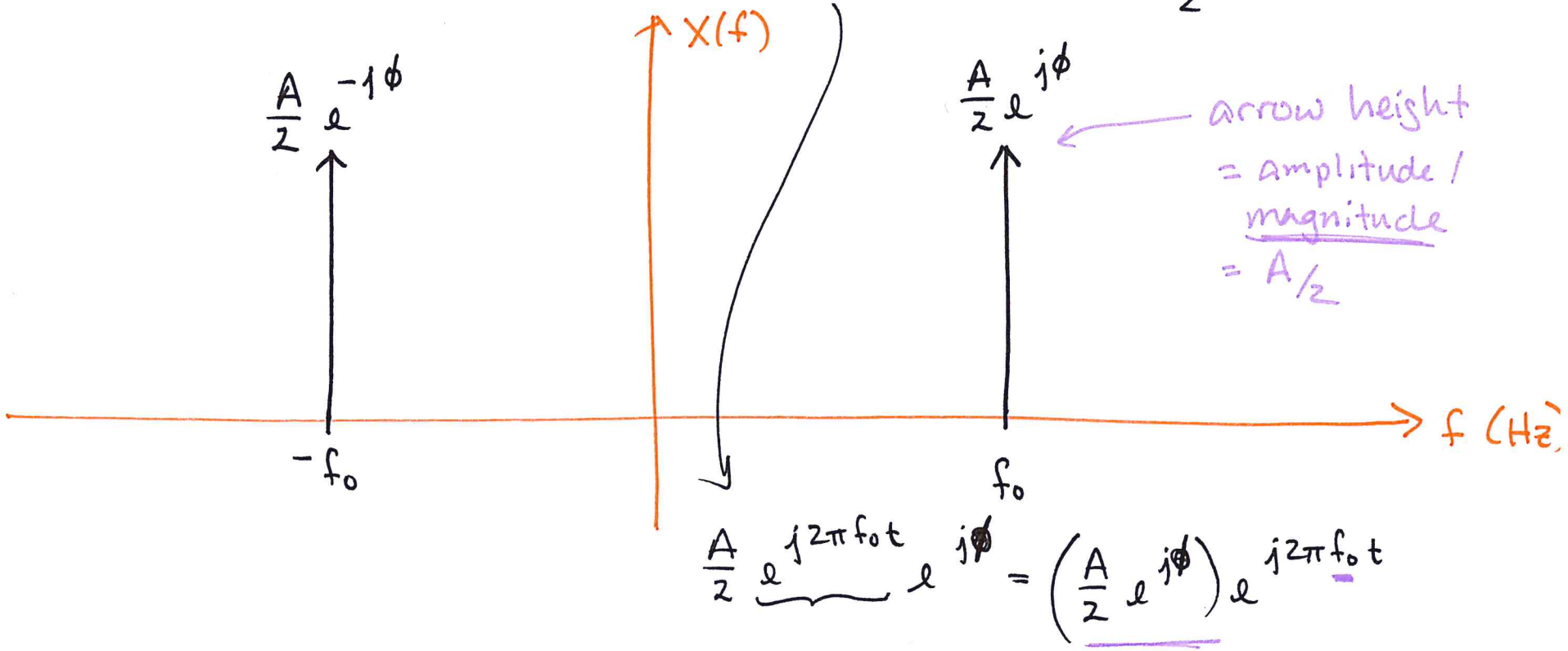
$$\mathrm{Re}\{e^{j\theta}\} = \cos\theta = \frac{e^{j\theta}}{2} + \frac{e^{-j\theta}}{2}$$

$$\mathrm{Im}\{e^{j\theta}\} = \sin\theta = \frac{e^{j\theta}}{2j} - \frac{e^{-j\theta}}{2j}$$

cos = sum of two complex exponentials, one has + frequency, one has - frequency and amplitude of each is half original amplitude.

Plotting a signal in frequency = spectrum = $X(f)$

$$x(t) = A\cos(2\pi f_0 t + \phi) = \frac{A}{2} e^{j(2\pi f_0 t + \phi)} + \frac{A}{2} e^{-j(2\pi f_0 t + \phi)}$$



Spectrum of a sum of sinusoids:

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

f_k = freq of k^{th} cos
ϕ_k = phase " " "
A_k = amplitude " " "

$$= A_0 + \sum_{k=1}^{N} \left[\frac{A_k}{2} e^{j(2\pi f_k t + \phi_k)} + \frac{A_k}{2} e^{-j(2\pi f_k t + \phi_k)} \right]$$

$$= \underbrace{A_0}_{a_0} + \sum_{k=1}^{N} \left[\underbrace{\frac{A_k}{2} e^{j\phi_k}}_{a_k} e^{j2\pi f_k t} + \underbrace{\frac{A_k}{2} e^{-j\phi_k}}_{a_k^*} e^{-j2\pi f_k t} \right]$$

$$= a_0 + \sum_{k=1}^{N} a_k e^{j2\pi f_k t} + a_k^* e^{-j2\pi f_k t}$$

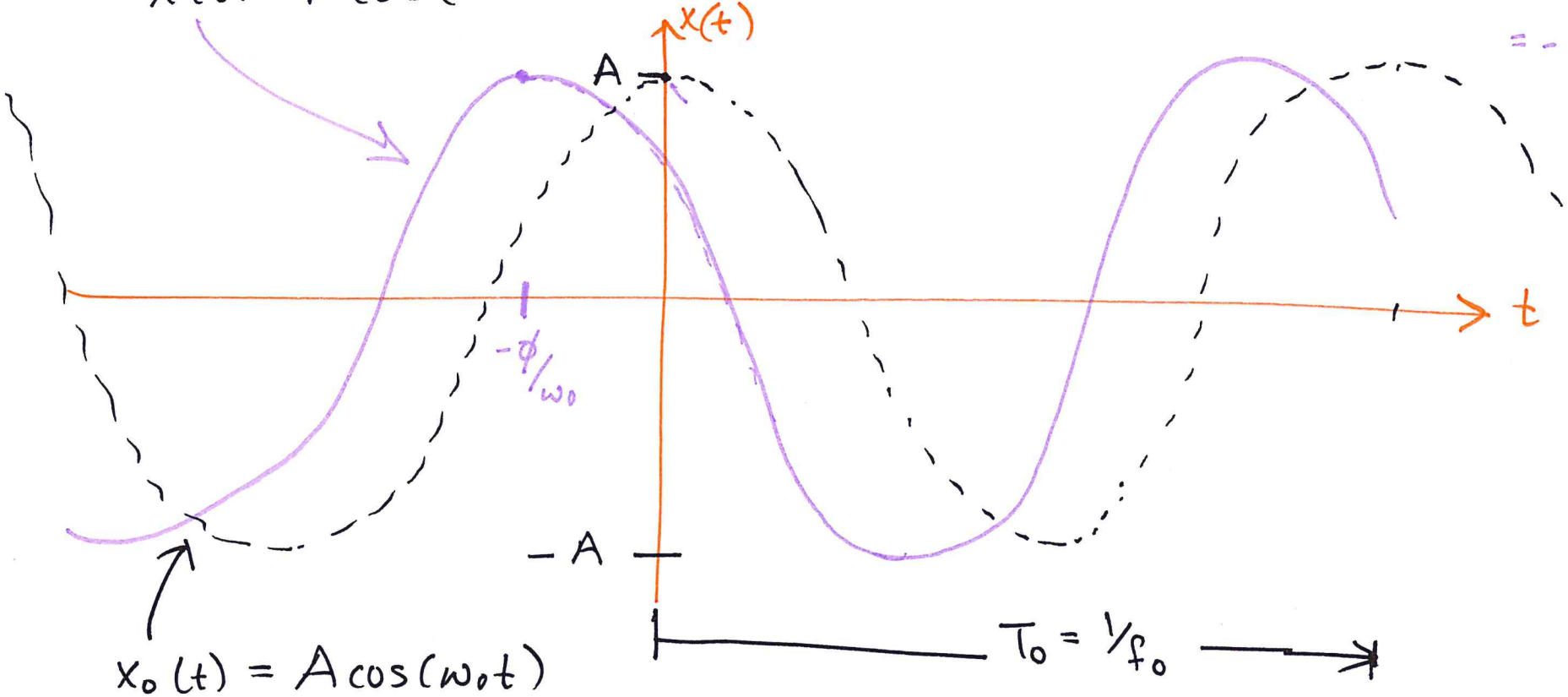
Plotting a signal ($ in time)

$$x(t) = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

$\omega_0 t + \phi = 0$

$t = -\phi/\omega_0$

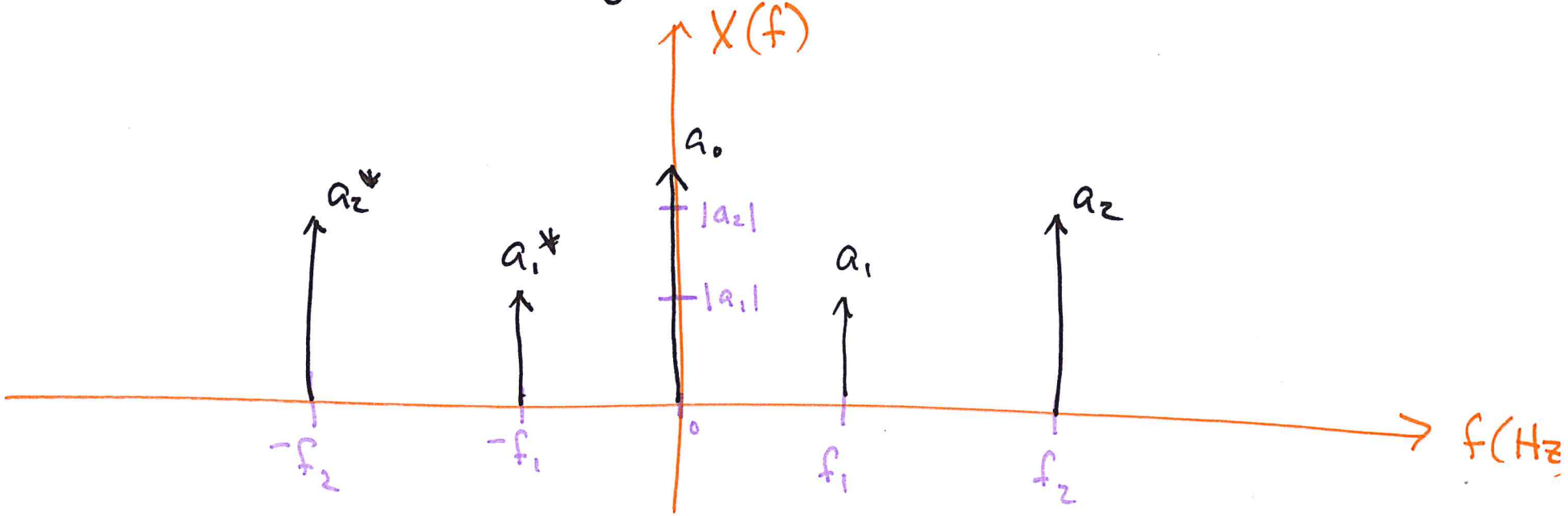
$= -\phi/2\pi f_0$



(Two-sided) spectrum of $x(t)$

$$\{ (0, a_0), (f_1, a_1), (-f_1, a_1^*), \ldots, (f_N, a_N), (-f_N, a_N^*) \}$$

= list of frequencies & amplitudes in $x(t)$



$\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$ $\cos(0) = 1$

Example 1:

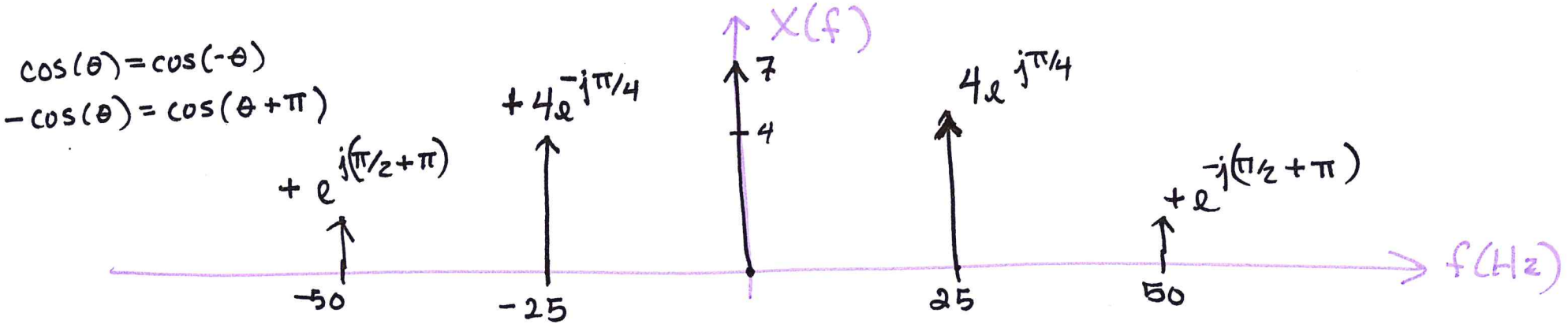
$$x(t) = 7 + 8\cos(50\pi t + \pi/4) - 2\cos(100\pi t - \pi/2)$$

$$= 7 + \frac{8}{2}e^{j(50\pi t + \pi/4)} + \frac{8}{2}e^{-j(50\pi t + \pi/4)} - \frac{2}{2}e^{j(100\pi t - \pi/2)} - \frac{2}{2}e^{-j(100\pi t - \pi/2)}$$

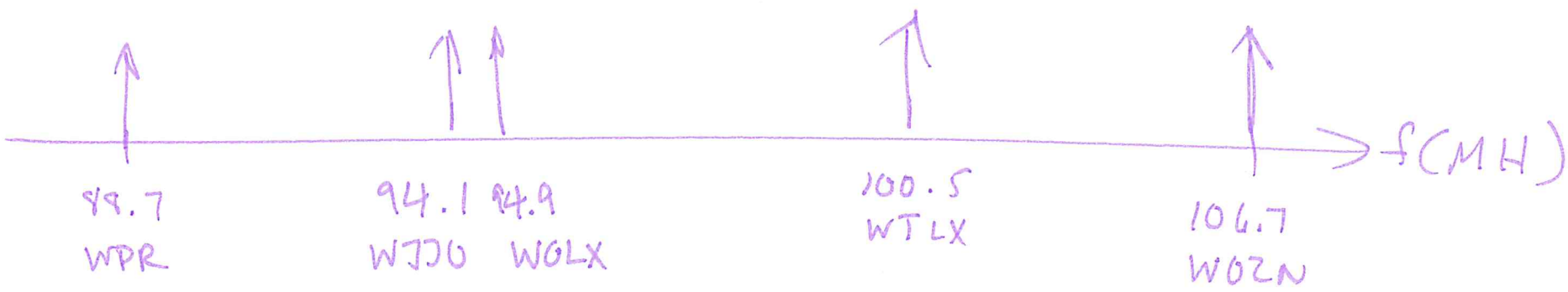
($f_k \cdot 2\pi t$)

$\cos(\theta) = \cos(-\theta)$

$-\cos(\theta) = \cos(\theta + \pi)$



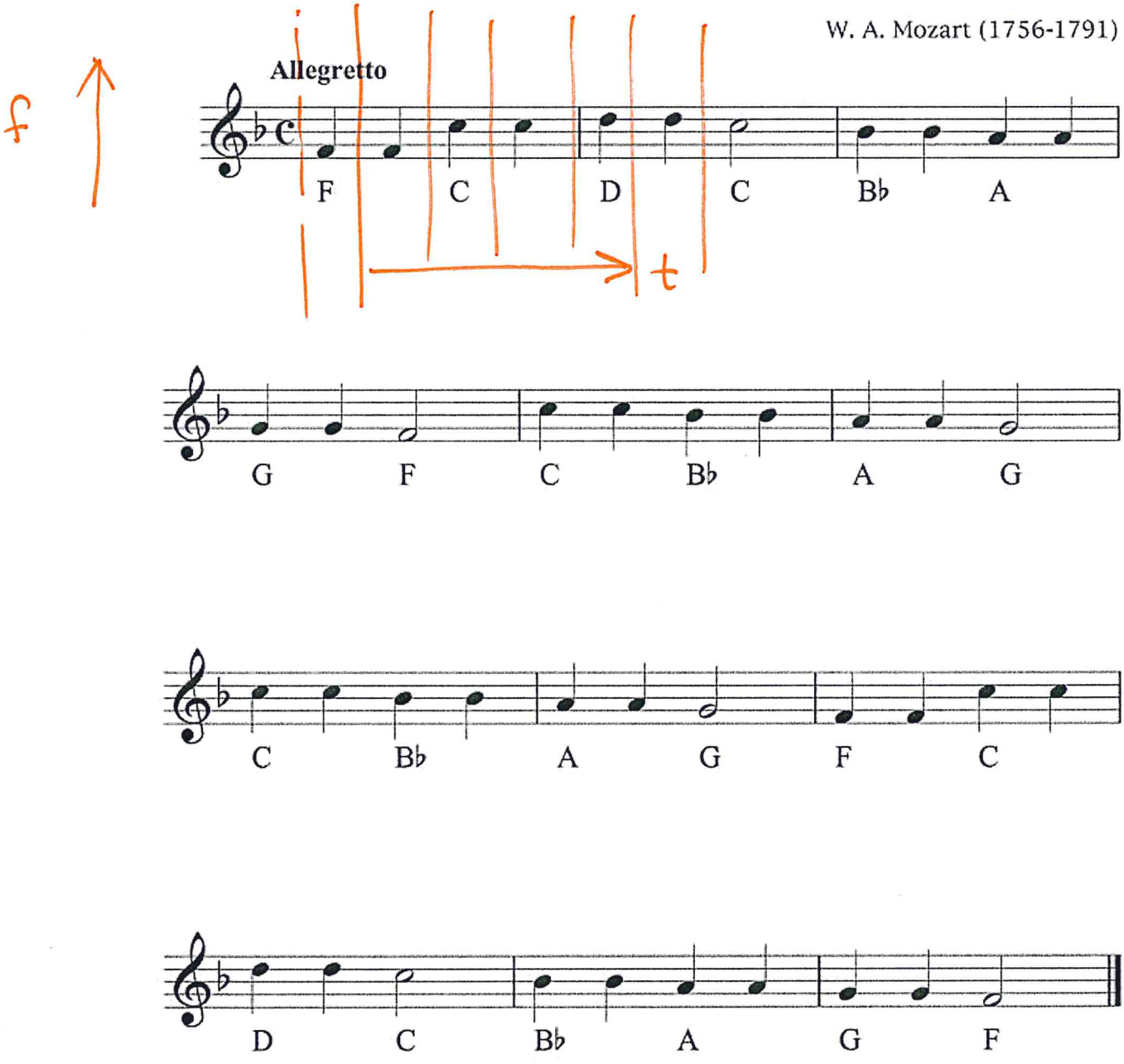
FM Spectrum



Twinkle, Twinkle, Little Star

very easy arrangements
created and edited by
Fabrizio Ferrari

W. A. Mozart (1756-1791)



Example 2:

$$x(t) = 7 + 8\cos(50\pi t + \pi/4) - 2\cos(50\pi t - \pi/2)$$

$$= \mathrm{Re}\left\{7 + 8e^{j(50\pi t + \pi/4)} - 2e^{j(50\pi t - \pi/2)}\right\}$$

$$= \mathrm{Re}\left\{7 + e^{j50\pi t}\left[\underbrace{8e^{j\pi/4} - 2e^{-j\pi/2}}_{= 9.5e^{j0.3\pi}}\right]\right\}$$

$$= 7 + 9.5\cos(50\pi t + 0.3\pi)$$