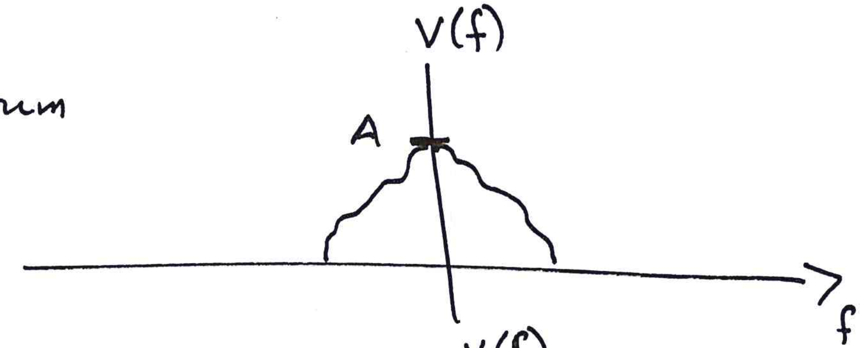
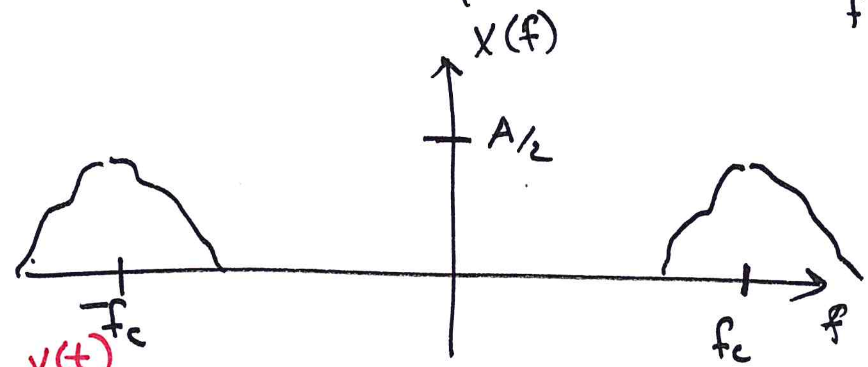
Recap of AM communications

auditory signal : $v(t) \longrightarrow$ spectrum



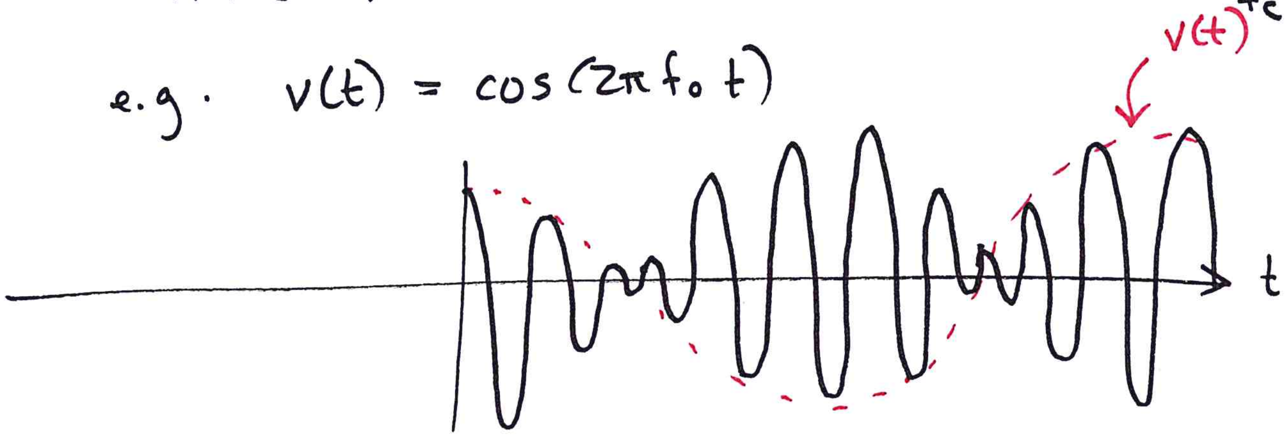
carrier signal : $\cos(2\pi f_c t)$

f_c = carrier frequency



transmit $x(t) = v(t) \cdot \cos(2\pi f_c t)$

e.g. $v(t) = \cos(2\pi f_0 t)$



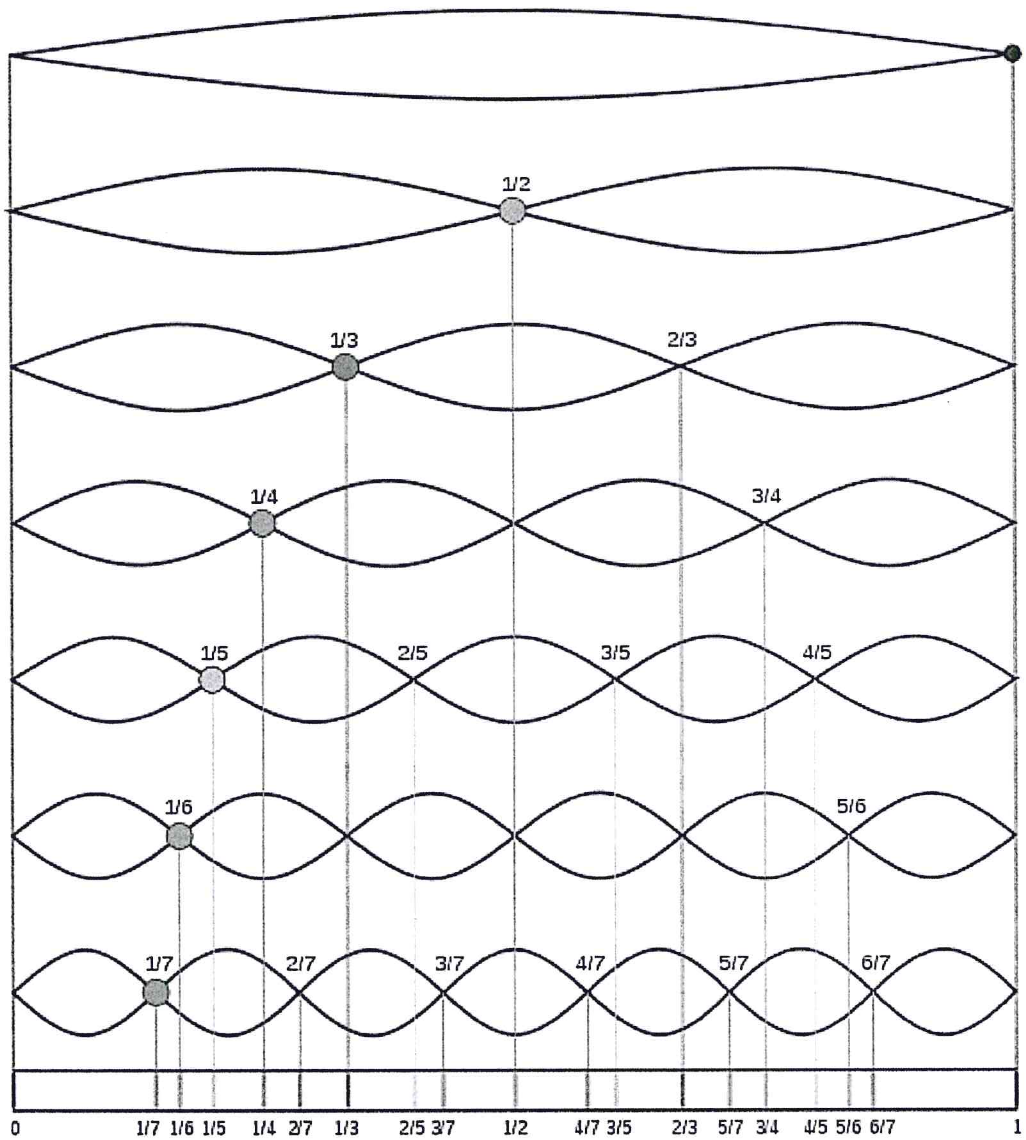
Lecture 6: Harmonics + Periodicity

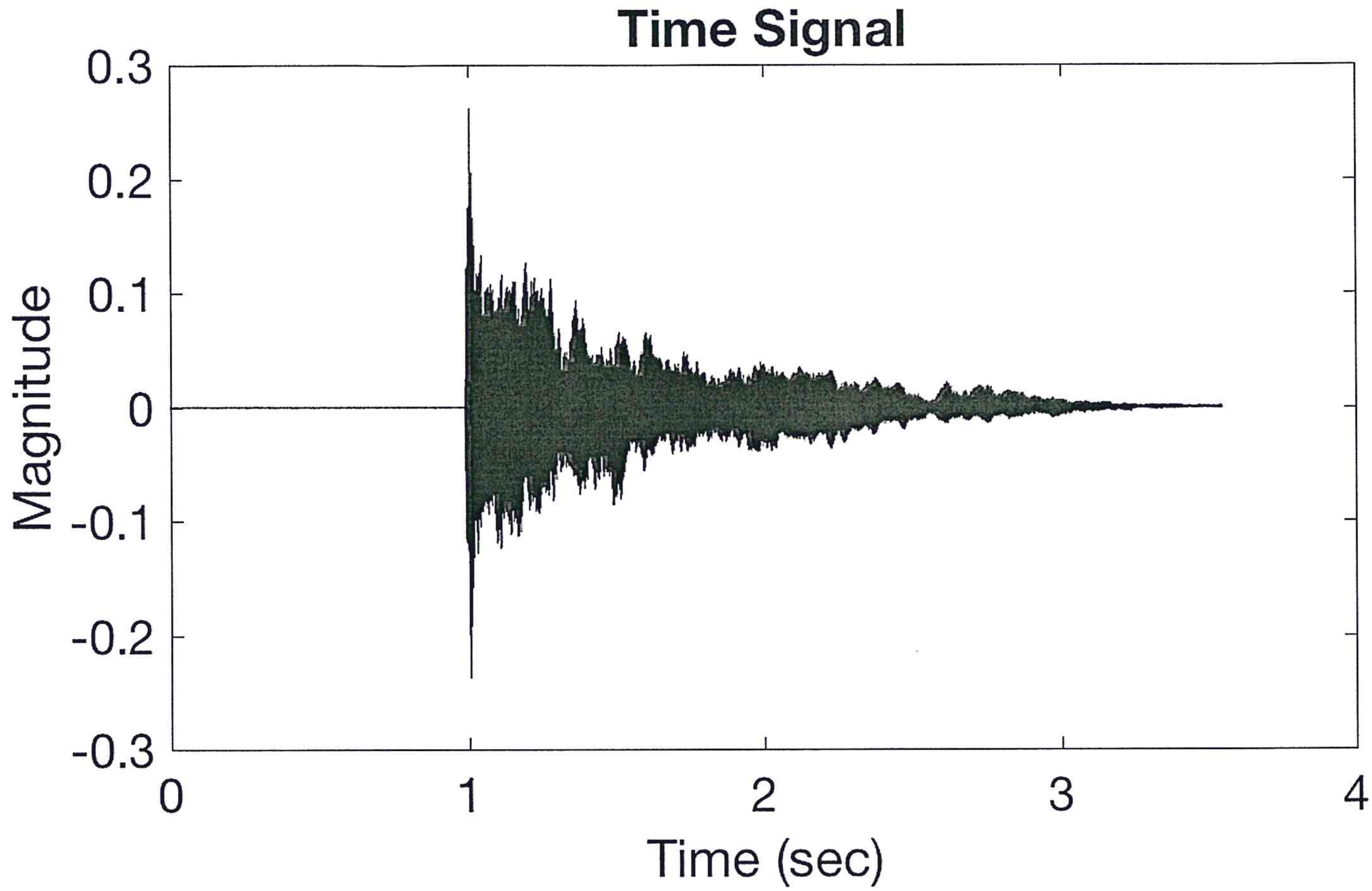
Let $x(t) = \cos(2\pi f_0 t)$

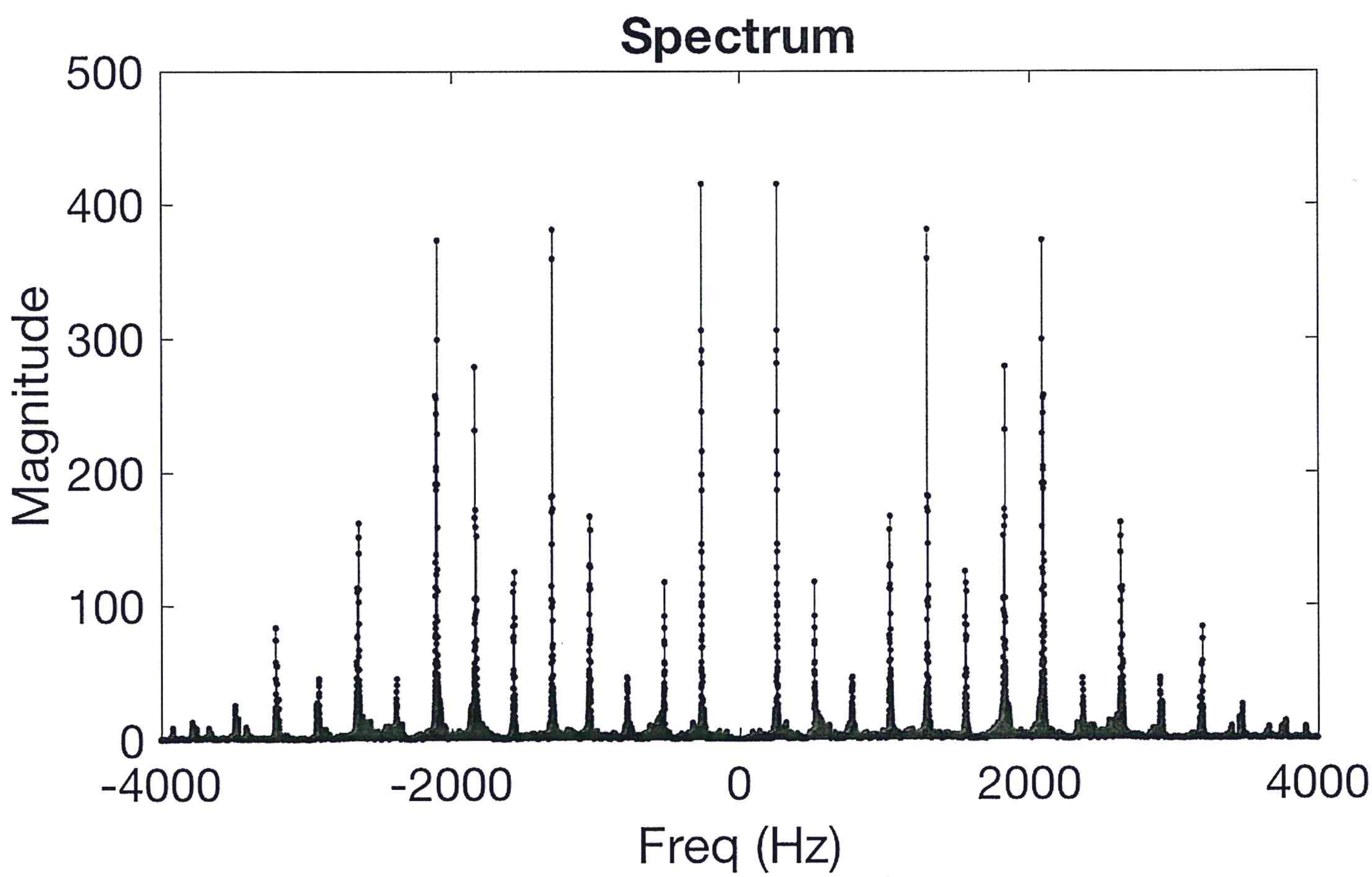
Harmonic frequencies (of f_0) : $2f_0, 3f_0, 4f_0, \ldots$

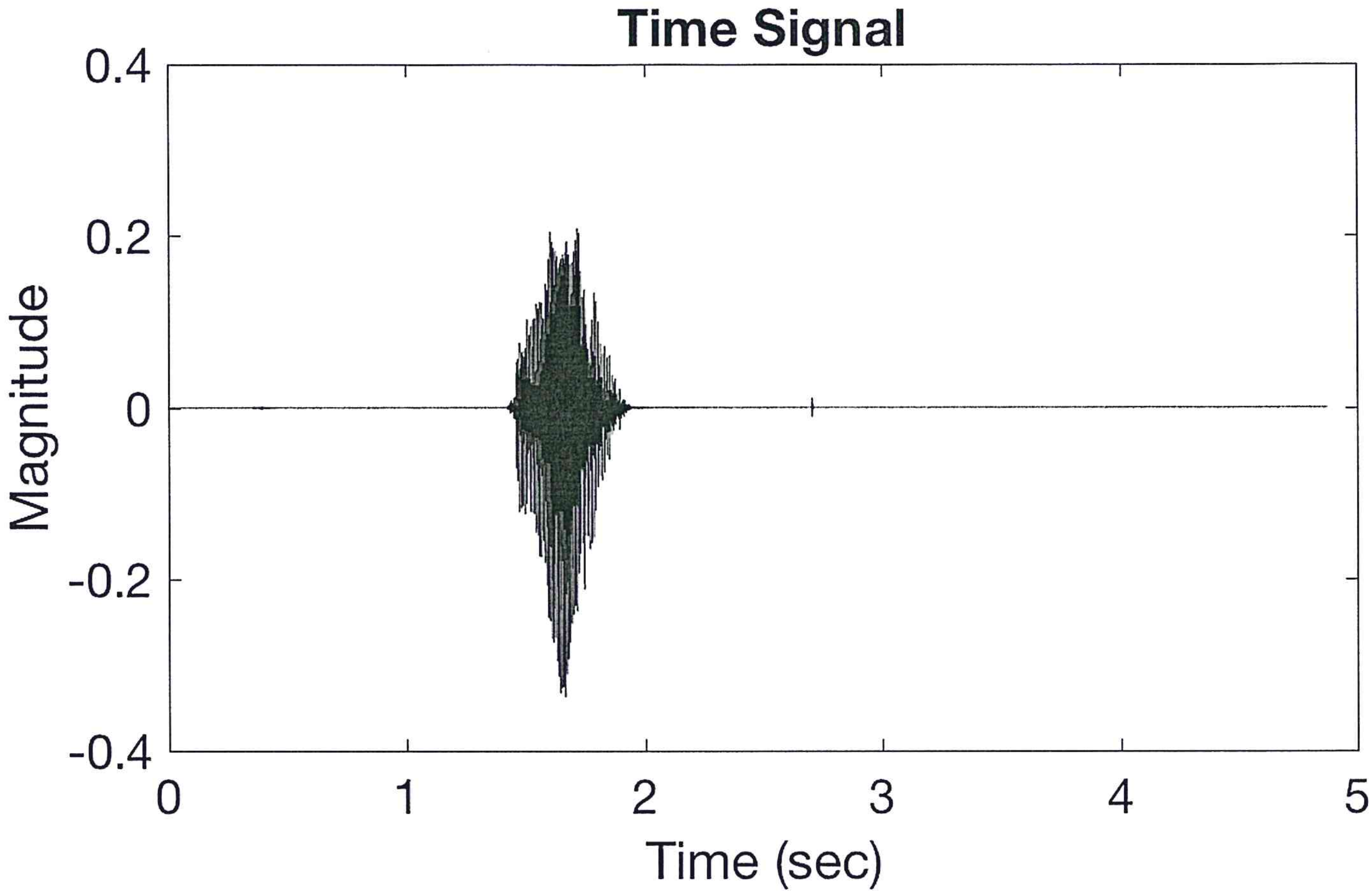
Harmonics (of signal $x(t)$) : $\cos(2\pi(2f_0)t), \cos(2\pi(3f_0)t), \cos(2\pi(4f_0)t), \ldots$

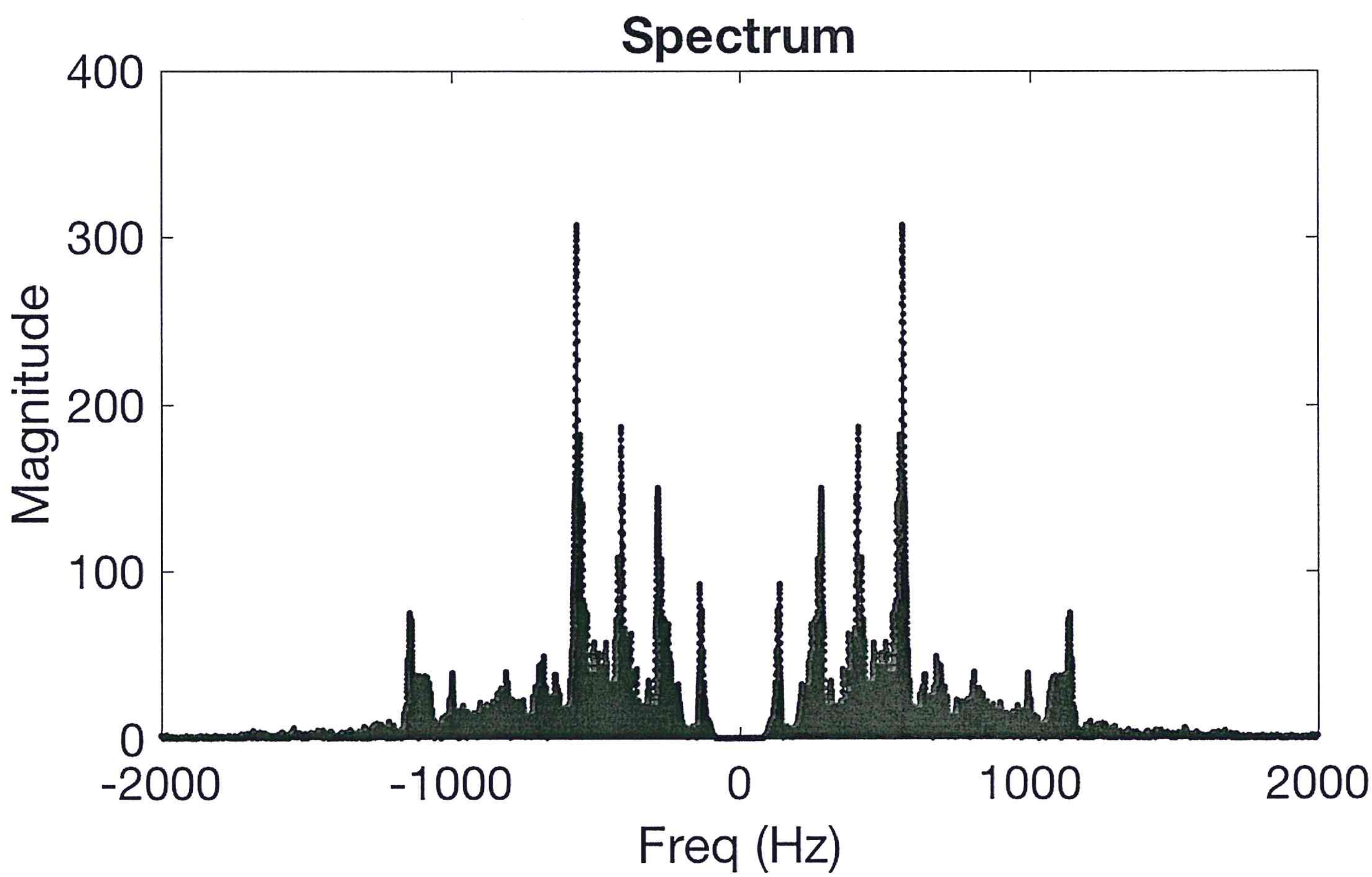
Any periodic signal can be represented by a sum of sinusoids that are harmonically related!


1/2
1/3
2/3
1/4
3/4
1/5
2/5
3/5
4/5
1/6
5/6
1/7
2/7
3/7
4/7
5/7
6/7
0
1/7 1/6 1/5 1/4 2/7 1/3 2/5 3/7 1/2 4/7 3/5 2/3 5/7 3/4 4/5 5/6 6/7
1


Time Signal
Magnitude
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
1
2
3
4
Time (sec)


Spectrum
Magnitude
500
400
300
200
100
0
-4000
-2000
0
2000
4000
Freq (Hz)


Time Signal
0.4
0.2
0
-0.2
-0.4
Magnitude
0
1
2
3
4
5
Time (sec)


Spectrum
400
300
200
100
0
Magnitude
-2000
-1000
0
1000
2000
Freq (Hz)

Periodic signals:

A signal is periodic with period T_0 seconds if $x(t) = x(t+T_0)$ for all t.

Ex 1 Sinusoid $x(t) = \cos(2\pi f_0 t + \phi)$

$x(t)$ is periodic with period $T_0 = 1/f_0$

verify: $x(t+T_0) = \cos(2\pi f_0 (t+T_0) + \phi)$

$= \cos(2\pi f_0 t + 2\pi f_0 T_0 + \phi)$

$= \cos(2\pi f_0 t + 2\pi + \phi)$

$= \cos(2\pi f_0 t + \phi) = x(t)$

$\boxed{\cos(\theta + 2\pi) = \cos(\theta)}$

$\cos(\theta + 2\pi k) = \cos(\theta)$ for any integer k

Ex 2 : Harmonic signal

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi \underline{(kf_0)} t + \phi_k)$$

$x(t)$ is periodic with period $T_0 = 1/f_0$

verify: $x(t+T_0) = \sum_{k=1}^{N} A_k \cos(2\pi k f_0 (t+T_0) + \phi_k)$

$$= \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \underbrace{2\pi k f_0 T_0}_{=2\pi k} + \phi_k)$$

$$= \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \phi_k) = x(t) \quad \checkmark$$

Given a signal with multiple sinusoidal components, what is the fundamental frequency?

Formally, given $x(t)$, what is the smallest T_0 such that $x(t) = x(t+T_0)$ for all t? Then $\underbrace{f_0 = 1/T_0}_{\text{"fundamental frequency"}}$.

Ex. $x(t) = \underbrace{3\cos(2\pi t + \pi/4)}_{\substack{f_1 = 1 \text{ Hz} \\ T_1 = 1/f_1 = 1 \text{ sec}}} - \underbrace{\cos(5\pi t + \pi/7)}_{\substack{f_2 = 2.5 \text{ Hz} \\ T_2 = 1/f_2 = 0.4 \text{ sec.}}}$

Find the smallest T_0 such that T_0 is an integer multiple of T_1 & T_2.

$T_1 \cdot 2 = 2$ sec. $\qquad T_2 \cdot 5 = 2$ sec. $\qquad \Rightarrow \underbrace{T_0 = 2 \text{ sec}}_{\text{not always an integer.}}$

integers!

$\Rightarrow f_0 = 1/T_0 = .5$ Hz.

When are signals aperiodic?

When they are not periodic!

When there is no $T_0 \neq 0$ such that $x(t) = x(t+T_0)$ for all t.

Ex. $x(t) = \underbrace{3\cos(2\pi t)}_{\substack{f_1 = 1\text{ Hz} \\ T_1 = 1/f_1 = 1\text{ sec}}} - \underbrace{\cos(5t)}_{\substack{f_2 = 5/2\pi\text{ Hz} \\ T_2 = 2\pi/5\text{ sec.}}}$

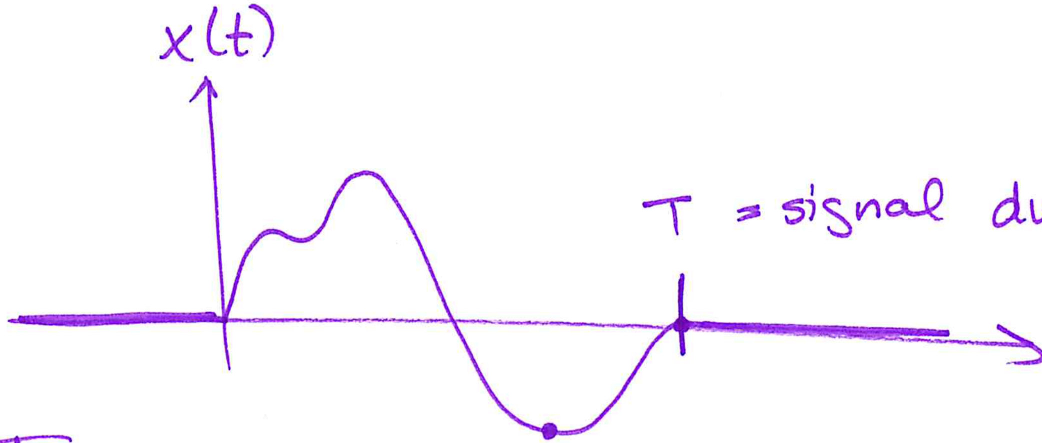
no π

there are no integers m and n such that

$$m \cdot T_1 = n \cdot T_2$$

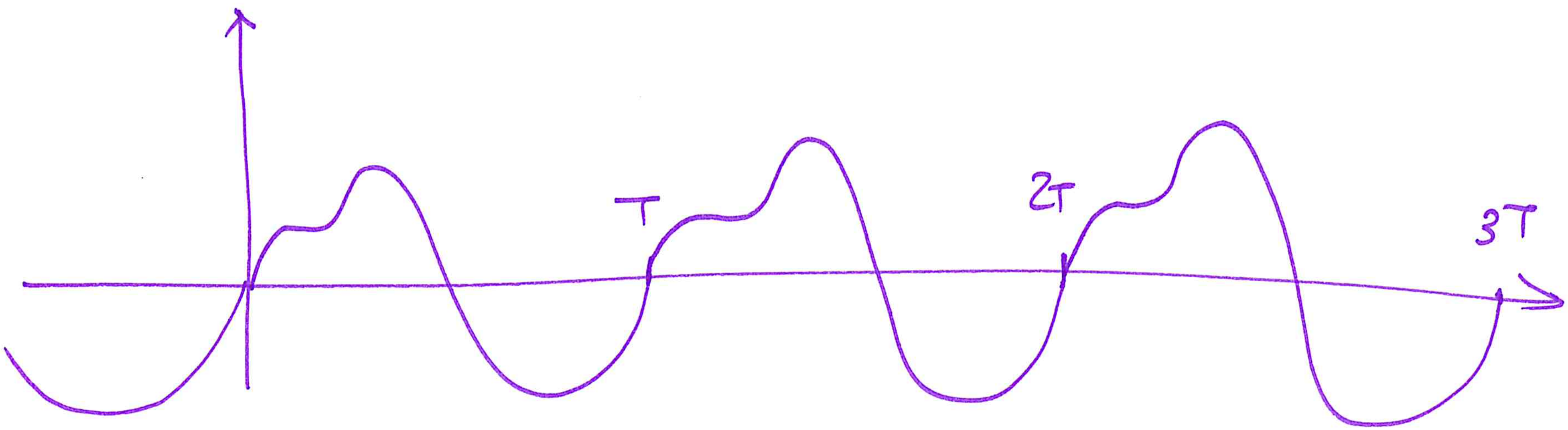
Finite duration signals:

periodic signal
$x(t) = x(t+T_0)$ for all t



T = signal duration

Trick: view $x(t)$ as one period of a periodic signal



Signals that we store on the computer are of finite duration, BUT we can treat them as periodic if it's mathematically convenient.