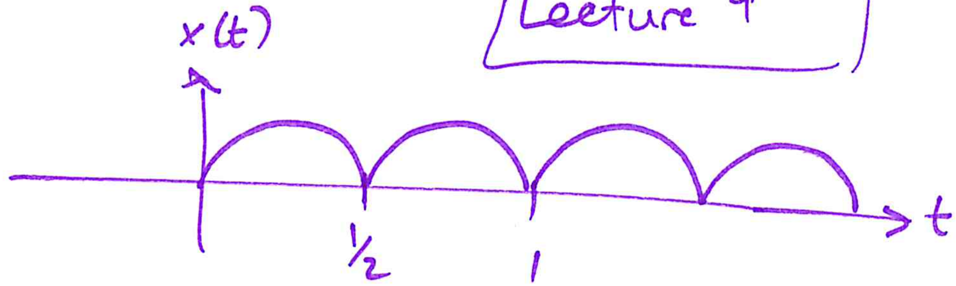
Example from last time:



$$x(t) = |\sin(2\pi t)|$$

$T_0 = 1/2$ s.

$f_0 = 1/T_0 = 2$ Hz.

$j^2 = -1$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k f_0 t}\, dt$$

$$= \int_0^{1/2}\left[\frac{1}{j} e^{j2\pi t(1-2k)} - \frac{1}{j} e^{-j2\pi t(1+2k)}\right] dt.$$

$$= \frac{1}{j}\left.\frac{e^{j2\pi t(1-2k)}}{j2\pi(\underline{1-2k})}\right|_0^{1/2} + \frac{1}{j}\left.\frac{e^{-j2\pi t(1+2k)}}{+j2\pi(\underline{1+2k})}\right|_0^{1/2}$$

$$* = \left(\frac{-e^{j\pi(1-2k)}}{2\pi(1-2k)}\right) - \left(\frac{-1}{2\pi(1-2k)}\right) + \left(\frac{-e^{-j\pi(1+2k)}}{2\pi(1+2k)}\right) - \left(\frac{-1}{2\pi(1+2k)}\right)$$

Recall: $e^{j\pi} = -1 \qquad e^{-j2\pi k} = 1 \qquad e^{-j\pi} = \frac{1}{e^{j\pi}} = \frac{1}{-1} = -1$

$$* = \frac{-e^{j\pi} \cdot e^{-j\pi 2k}}{2\pi(1-2k)} + \frac{1}{2\pi(1-2k)} - \frac{e^{-j\pi} e^{-j\pi 2k}}{2\pi(1+2k)} + \frac{1}{2\pi(1+2k)}$$

$$= \frac{-(-1)(1)}{2\pi(1-2k)} + \frac{1}{2\pi(1-2k)} - \frac{(-1)(1)}{2\pi(1+2k)} + \frac{1}{2\pi(1+2k)}$$

$$\boxed{a_k = \frac{1}{\pi(1-2k)} + \frac{1}{\pi(1+2k)}}$$

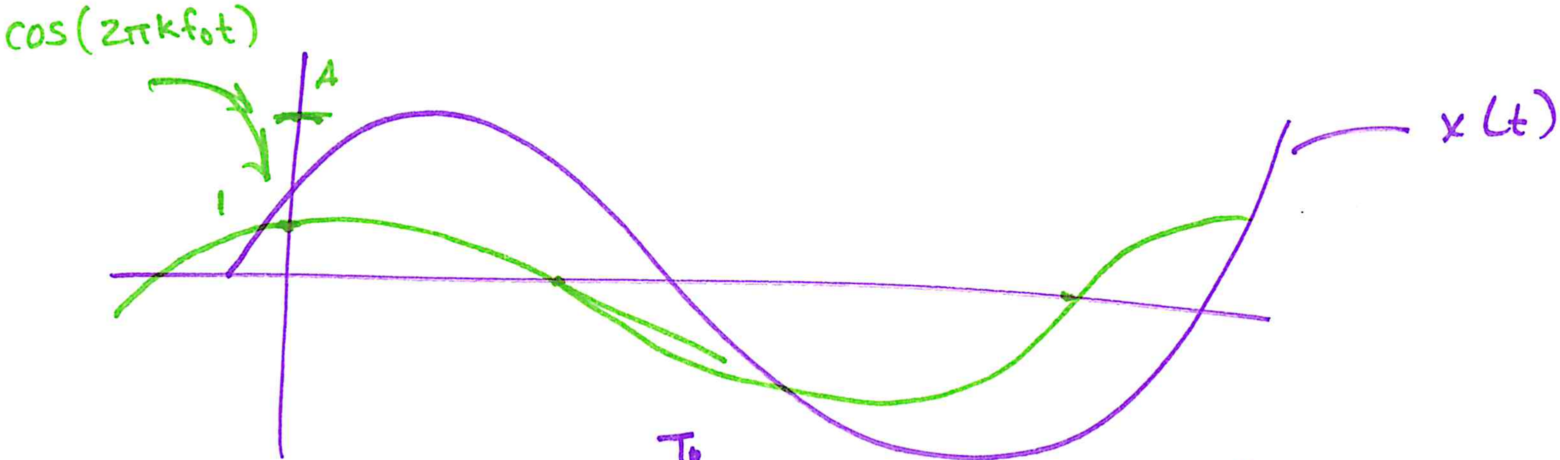
$e^{j\theta} = \cos(\theta) + j\sin\theta$

$e^{j(-2\pi k)} = \cos(-2\pi k) + j\sin(-2\pi k) = 1$

Note:

we set $\underline{a_k} = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k f_0 t}\, dt$ ⊛

the sinusoid $a_k \cdot e^{j2\pi k f_0 t}$ is the "best" fit to $x(t)$ @ freq kf_0



let $\text{error}(a) = \int_0^{T_0} \left| x(t) - a\, e^{j2\pi k f_0 t} \right|^2 dt$

we can show that a_k from ⊛ minimizes error(a)

$\text{error}(a_k) \leq \text{error}(a)$ for all a

Lecture 9

So far in class, continuous time / analog signals

signal $x(t)$ where t is real-valued

e.g. $x(t) = \cos(2\pi f t)$

In lab, we represent signals as list of numbers for storing and processing

Ex 1 CD audio: 3 min song $\approx$ 32 MB

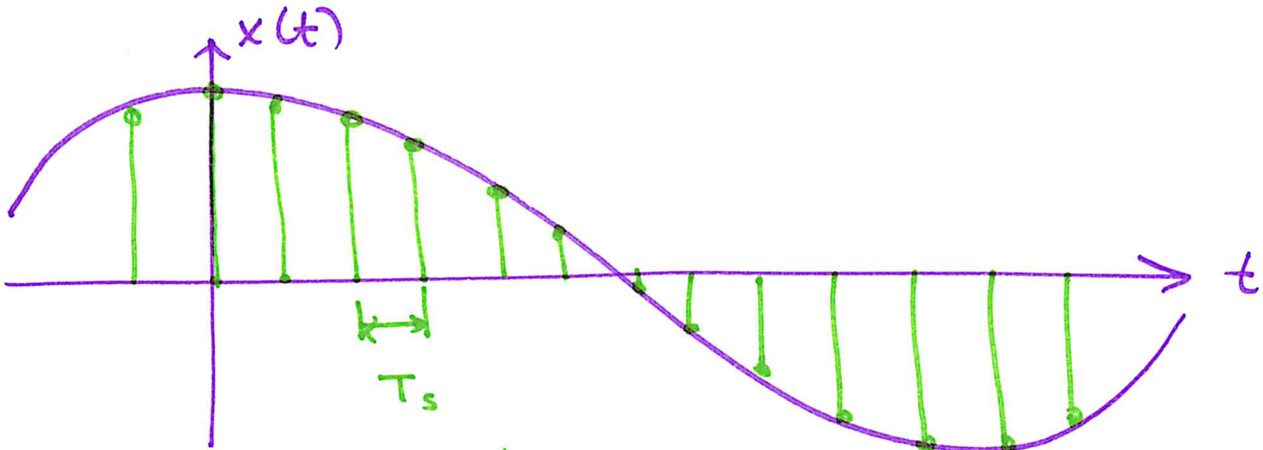
Ex 2 digital image — store individual pixels

Analog to Digital Conversion

convert a continous-time signal to a digital or discrete-time signal

AKA $\boxed{\text{"sampling"}}$

Basic idea:



= sampling period

$1/T_s = f_s$ = sampling frequency.

$$\underbrace{x[n]}_{n^{th}\text{ discrete-time sample}} = x(n \cdot T_s)$$

for $n = \ldots, -2, -1, 0, 1, 2, \ldots$

Ex CD Audio:

$f_s = 44.1$ kHz $\Rightarrow$ 44,100 samples / sec

× 16 bits / sample ←

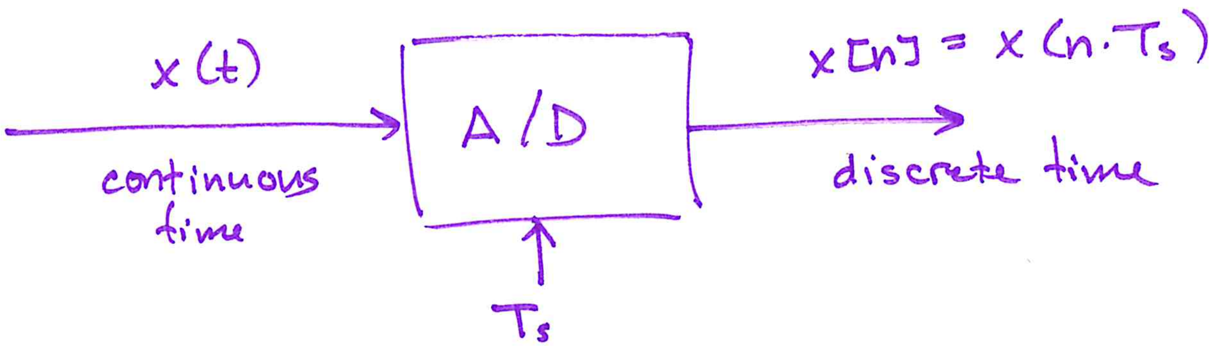
× 2 channels

× 60 sec / min

× 3 min

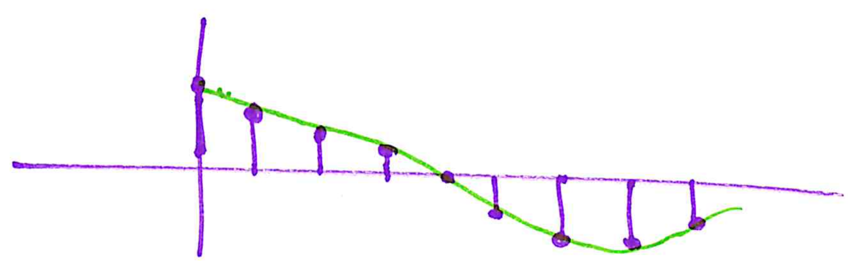
$\approx$ 254 Mbits = 32 MB

Block diagram



Also need to go from discrete time to continuous time

"connecting the dots"

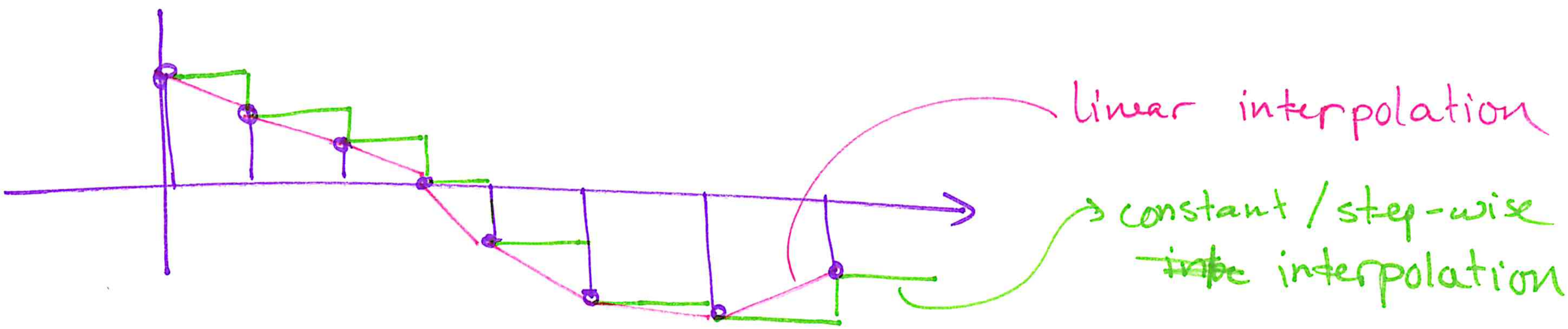


$$x[n] \longrightarrow \boxed{D/A} \longrightarrow y(t)$$

$\uparrow$ T_s

$\boxed{\text{want } y(t) = x(t)}$

How do we connect the dots? | $\boxed{\text{"Signal reconstruction"}}$



errors between $x(t)$ and $y(t)$ get smaller as T_s gets smaller – **BUT** small T_s → more bits → more storage

→ requires more $