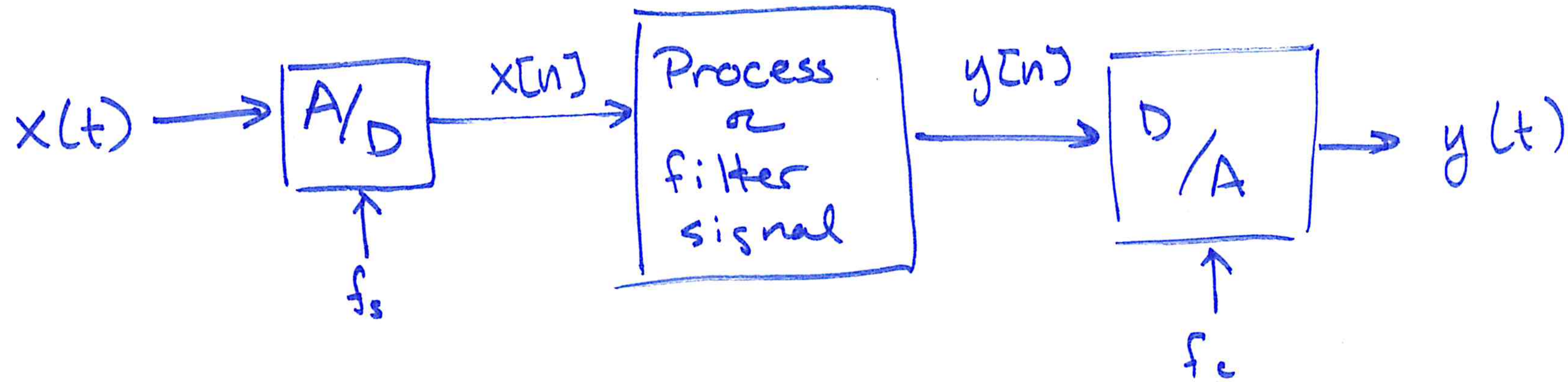
Lecture 15: Digital Signal Processing Systems



Ex: $x(t)$ = music

$x[n]$ = sampled music

filter = dft + scaling (bass boost)

$y[n]$ = samples w/ boosted bass

$y(t)$ = music (played w/ soundsc)

Generally, a filter is a device or process for removing or modifying frequency components of a signal.

Simple examples of filters

$$y[n] = x[n]^3$$

$$y[n] = \mathrm{ifft}\left(\mathrm{fft}(x[n]) \;.\!*\; \mathrm{weights}\right) \Leftarrow$$

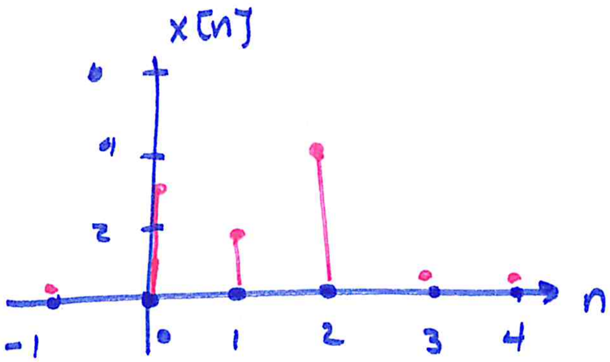
$$y[n] = \max\{x[n], x[n-1], \ldots, x[n-4]\}$$

$$y[n] = \frac{1}{2}x[n] + \frac{1}{3}x[n-1] + \frac{1}{5}x[n-2] \Leftarrow$$

Moving average filter : y[n] = average of past input samples.

e.g. $y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$

let



; what is y[n] ?

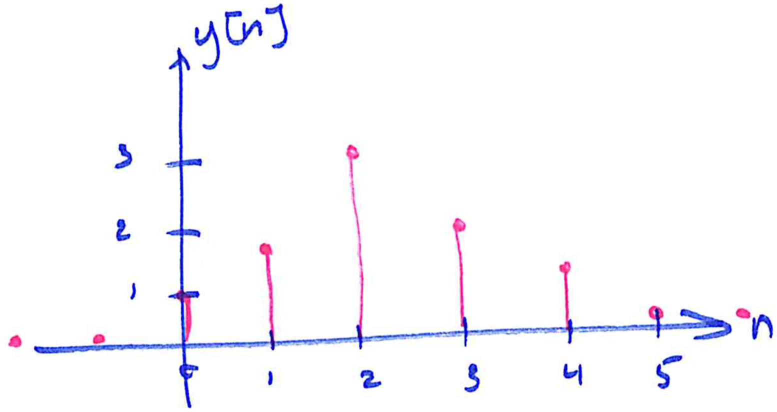
$y[0] = \frac{1}{3}x[0] + \frac{1}{3}x[-1] + \frac{1}{3}x[-2] = \frac{1}{3}\cdot 3 = 1$

$y[1] = \frac{1}{3}x[1] + \frac{1}{3}x[0] + \frac{1}{3}x[-1] = \frac{1}{3}(2) + \frac{1}{3}(3) = 1\,^{2}/_{3}$

$y[2] = \frac{1}{3}x[2] + \frac{1}{3}x[1] + \frac{1}{3}x[0] = \frac{1}{3}\cdot 9 = 3$

$y[3] = \frac{1}{3}x[3] + \frac{1}{3}x[2] + \frac{1}{3}x[1] = \frac{1}{3}(6) = 2$

$y[4] = \frac{1}{3}x[2] = 4/3$



Finite Impulse Response (FIR) filters

FIR filters = <u>weighted</u> moving average filter

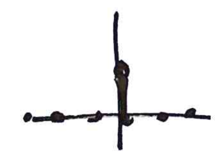
$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k] = b_0 \, x[n] + b_1 \, x[n-1] + b_2 \, x[n-2] + \cdots + b_M \, x[n-M]$$

e.g Moving average filter from pg 3,

$M = 2 \qquad b_0 = b_1 = b_2 = 1/3$

Impulse Response :

impulse signal (delta function) $\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$



impulse response of a filter is filter output if input is impulse signal

$$\delta[n] \longrightarrow \boxed{\text{FIR}} \longrightarrow \text{impulse response} = h[n]$$

An FIR filter is entirely characterized by its response to an impulse.

That is, if we know $h[n]$, then we can figure out how the filter will respond to any signal $x[n]$.

Ex: impulse response of moving average filter?

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$= \sum_{k=0}^{2} \tfrac{1}{3}\, x[n-k] \Rightarrow b_k = \begin{cases} \tfrac{1}{3} & \text{if } k=0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

to get impulse response, set $x[n] = \delta[n]$

$$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

In general, impulse response of FIR filter is

$$h[n] = \sum_{k=0}^{M} b_k \underbrace{\delta[n-k]}_{\substack{\text{only nonzero} \\ \text{for } k=n}} = \begin{cases} 0 & \text{if } n < 0 \\ b_0 & \text{if } n = 0 \\ b_1 & \text{if } n = 1 \\ b_2 & \text{if } n = 2 \\ \vdots & \\ b_M & \text{if } n = M \\ 0 & \text{if } n > M \end{cases}$$

$$\boxed{b_k = h[k] \text{ for } k = 0, 1, \ldots, M}$$

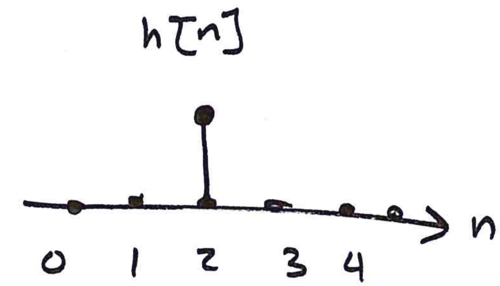
So we can calculate $\boxed{y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]}$

This type of sum is called a convolution of h and x.

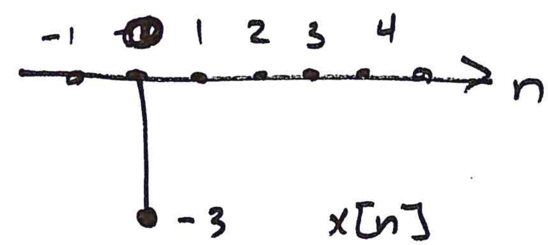
We often write this as $\boxed{y = h * x}$

Ex: let $h[n] = \begin{cases} 1 & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$

"delay filter"



let $x[n] = \begin{cases} -3 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases}$



Compute $y = h * x$.

only has value when k=0

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]\, x[0-k] = h[0] \cdot x[0] = 0 \cdot -3 = 0$$

only has value when k=1

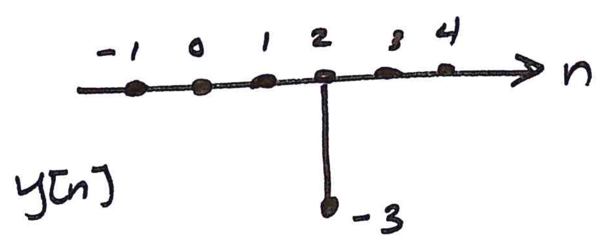
$$y[1] = \sum_{k=-\infty}^{\infty} h[k]\, x[1-k] = h[1] \cdot x[0] = 0$$

value when k=2

$$y[2] = \sum_{k=-\infty}^{\infty} h[k]\, x[2-k] = h[2] \cdot x[0] = 1 \cdot -3 = -3$$

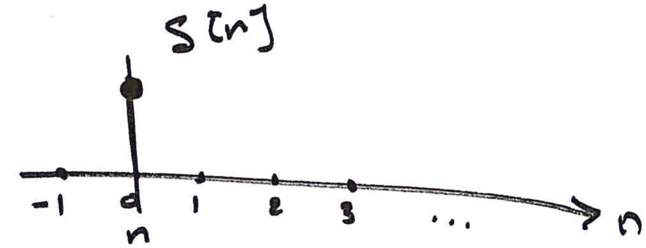
value when k

$$y[3] = \sum_{k=-\infty}^{\infty} h[k]\, x[3-k] = 0$$

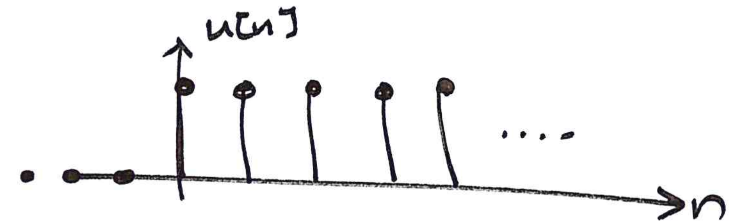


Two important discrete-time signals

Impulse : $\delta[n] = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases}$

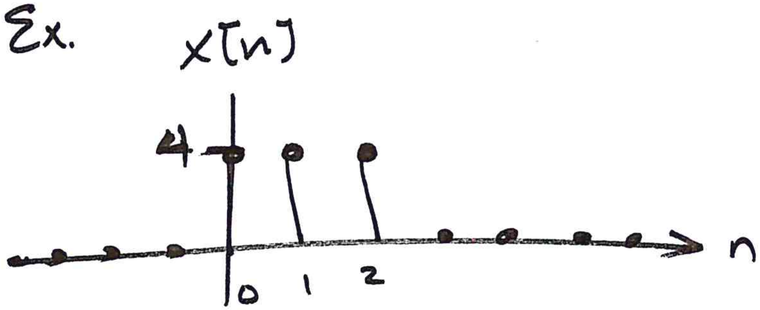


Step : $u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$



Impulse + Step functions are building blocks we can use to express ~~and~~ any signal.

Ex.



$$x[n] = 4\,\delta[n] + 4\,\delta[n-1] + 4\,\delta[n-2]$$

$$= 4\,u[n] - 4\,u[n-3]$$