Lecture 16

Signal Processing Systems:

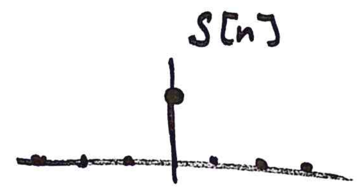
$$x(t) \longrightarrow \boxed{A/D} \xrightarrow{x[n]} \boxed{\text{filter}} \xrightarrow{y[n]} \boxed{D/A} \longrightarrow y(t)$$

Finite impulse response filter (FIR)

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

h is called the filter's "impulse response" because if

input $x[n] = \delta[n] = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n \neq 0 \end{cases}$



then $y[n] = h[n]$ and after some finite amount of time,

$y[n] = 0$

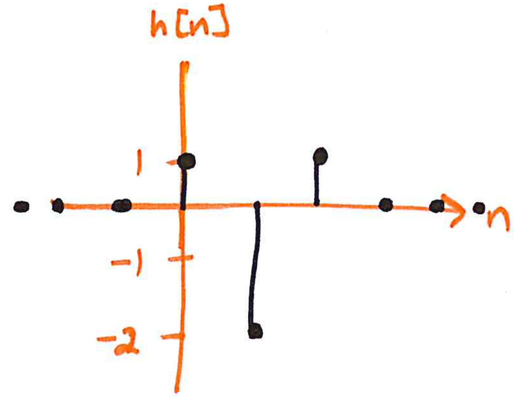
Can compute filter output via convolution

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

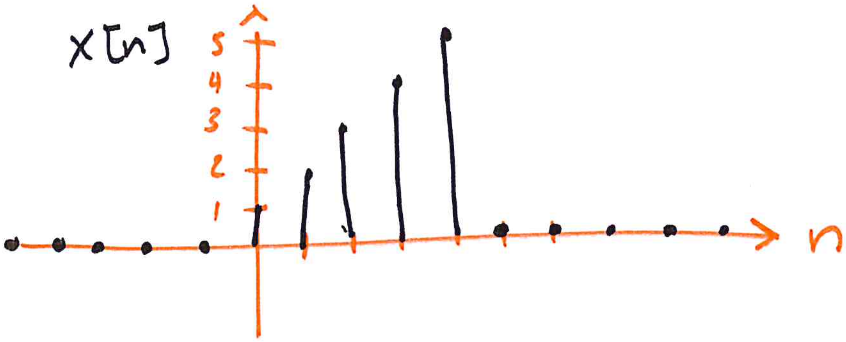
<u>not</u> multiplication

Example: discrete 2^{nd} derivative

$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$$



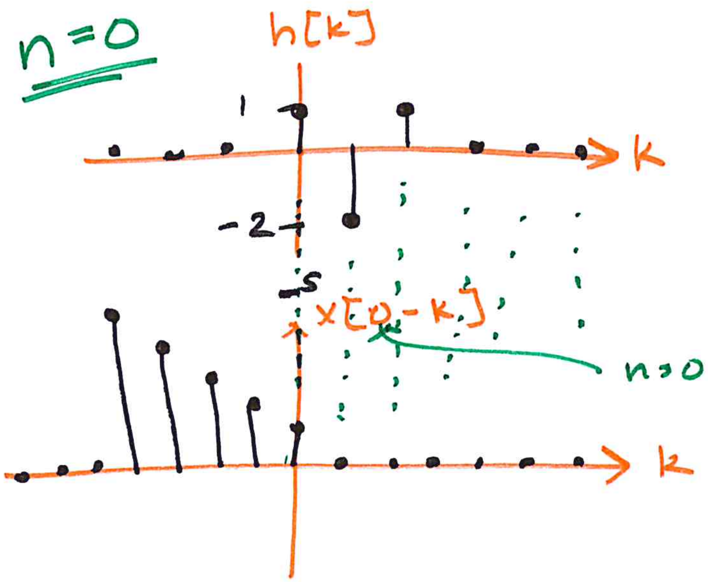
What filter output for input



What is $y = h * x$?

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$\underline{\underline{n=0}}$

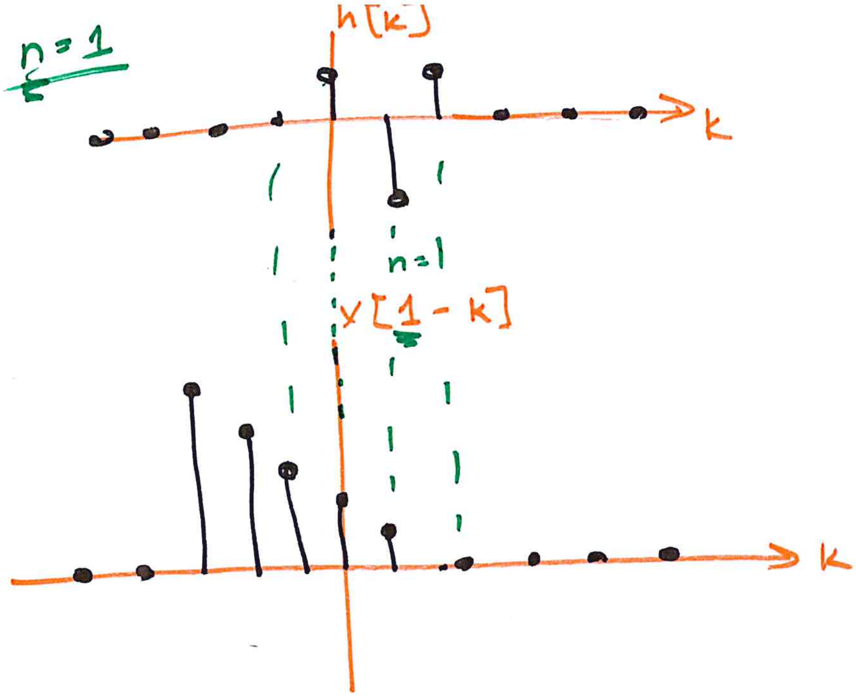


multiply and sum over all k

$$y[0] = \underbrace{1\cdot 1}_{k=0} + \underbrace{0\cdot(-2)}_{k=1} + \underbrace{0\cdot 1}_{k=2}$$

$$= \boxed{1}$$

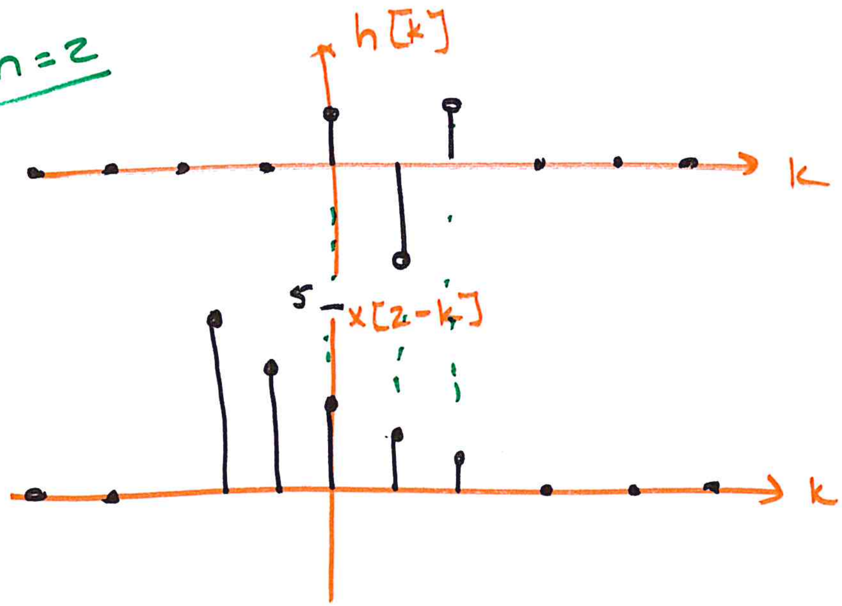
n = 1



$$y[1] = \underbrace{1 \cdot 2}_{k=0} + \underbrace{(-2) \cdot 1}_{k=1} + \underbrace{1 \cdot 0}_{k=2}$$

$$= 0$$

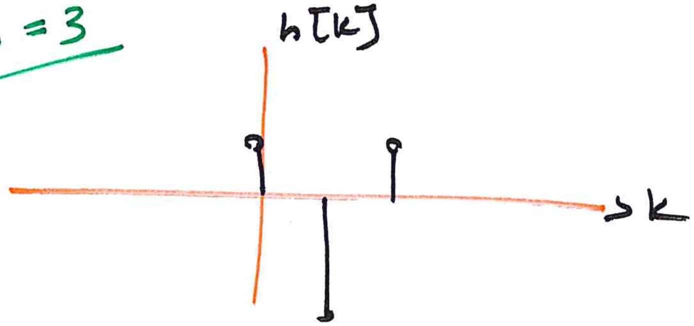
n = 2

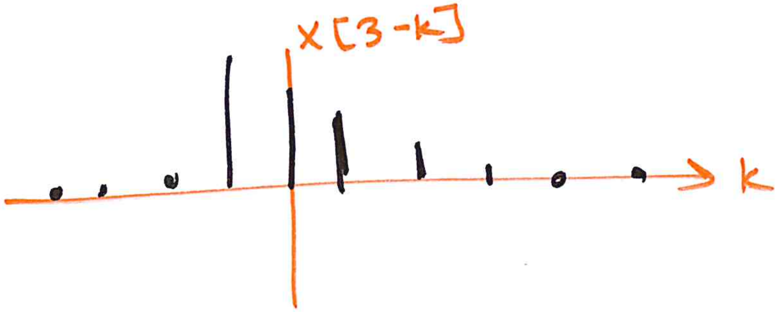


$$y[2] = \underbrace{1 \cdot 3}_{k=0} + \underbrace{(-2) \cdot 2}_{k=1} + \underbrace{1 \cdot 1}_{k=2}$$

$$= 0$$

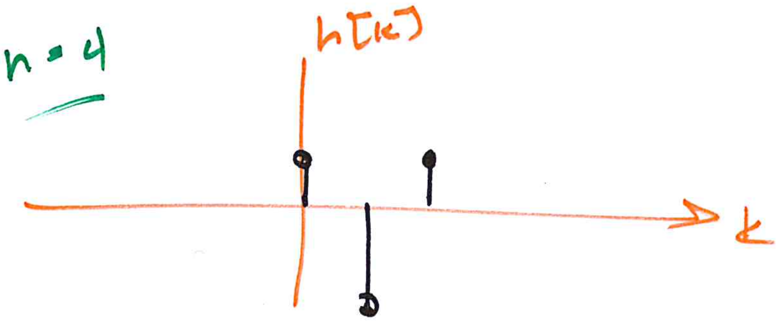
$n = 3$

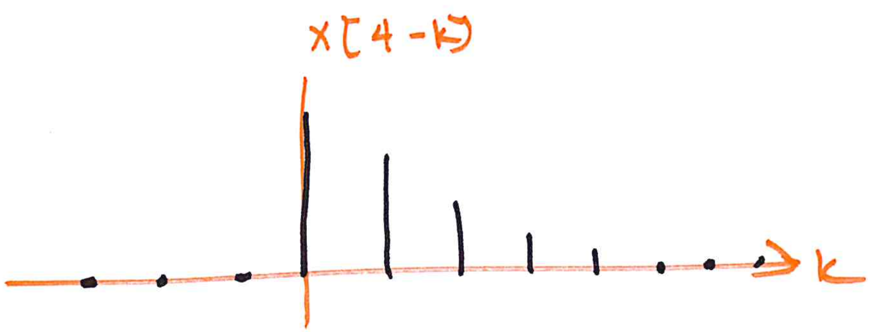




$$y[3] = 1\cdot 4 + (-2)3 + 1\cdot 2$$
$$= 0$$

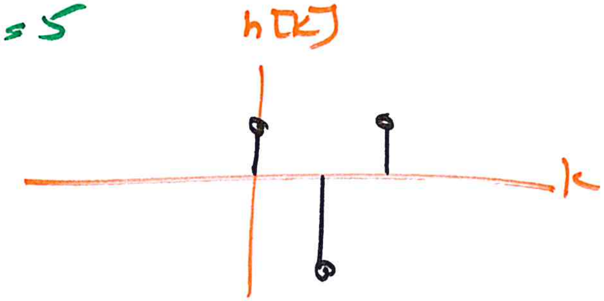
$n = 4$

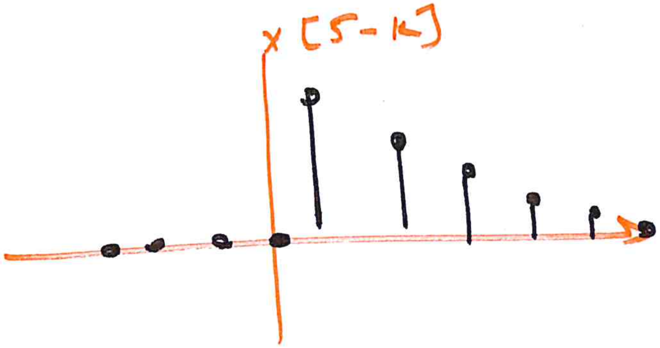




$$y[4] = 1\cdot 5 + (-2)4 + 1\cdot 3$$
$$= 0$$

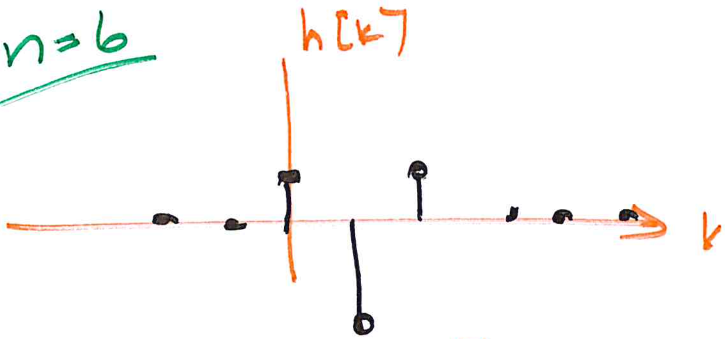
n=5

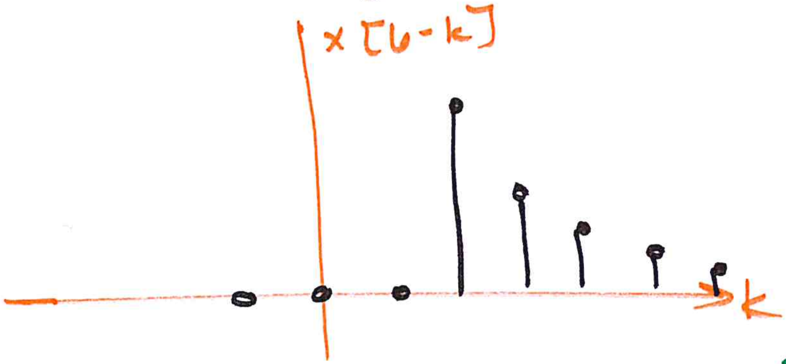




$$y[5] = 1\cdot 0 + (-2)\cdot 5 + 1\cdot 4$$
$$= -6$$

n=6





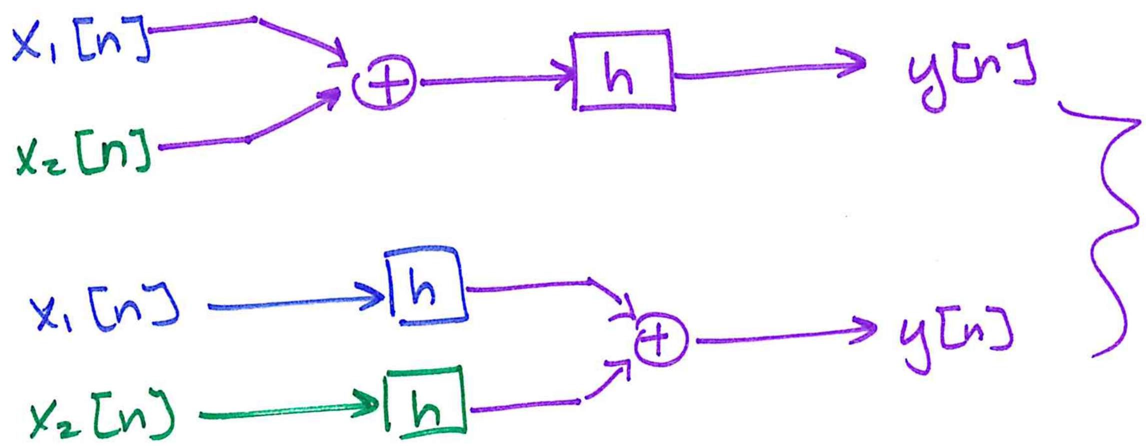
$$y[6] = 1\cdot 0 + (-2)\cdot 0 + 1\cdot 5$$
$$= 5$$

$$y = \dots [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad -6 \quad 5] \dots 0$$

Linear, Time-Invariant (LTI) & Causal filters. /systems

(all FIR filter are LTI)

Linear



for Linear filters/systems, these are <u>exactly</u> the same

if $\quad x_1[n] \longrightarrow \boxed{h} \longrightarrow y_1[n]$ and $x_2[n] \longrightarrow \boxed{h} \longrightarrow y_2[n]$

then $\quad a_1 \cdot x_1[n] + a_2 x_2[n] \longrightarrow \boxed{h} \longrightarrow a_1 y_1[n] + a_2 y_2[n]$

Ex 1 : $y[n] = x[n] + 3x[n-1]$

let $x[n] = a_1 x_1[n] + a_2 x_2[n]$

$$y[n] = (a_1 x_1[n] + \cancel{3} a_2 x_2[n]) + 3(a_1 x_1[n-1] + a_2 x_2[n-1])$$

$$= a_1 (x_1[n] + 3x_1[n-1]) + a_2 (x_2[n] + 3x_2[n-1])$$

$$= a_1 y_1[n] + a_2 y_2[n]$$

$\Longrightarrow$ linear !

Ex 2 : $y[n] = x^2[n]$

let $x_1[n] = 2$, $x_2[n] = 4$ for all n

$y_1[n] = 4$, $y_2[n] = 16$ for all n

$$x_1[n] + x_2[n] \longrightarrow \boxed{h} \longrightarrow (x_1[n] + x_2[n])^2 = (2+4)^2 = 36$$

but $y_1[n] + y_2[n] = 4 + 16 = 20$

AM radio

we receive $x[n] = \sum_{l=0}^{L} x_l[n]$ different radio stations