Last Lecture

How far you've come:

started with passing knowledge of trig, complex #'s.

now:

- Complex exponentials & sinusoids
- music synthesis
- spectra & spectrograms
- filtering fMRI data
- removing 60Hz noise from ECG
- AM radio
- Image deblurring
- JPEG compression
- aliasing!!!

Next steps:

330: Signals & Systems

431: Digital SP: sampling & recon, Z xform,
FIR & IIR design

532: Matrix Methods in ML

533: Image Processing

Frequency response of filters:

$$H(\hat{f}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j2\pi \hat{f} k}$$

if $x[n] = e^{j2\pi \hat{f} n} \rightarrow \boxed{h} \rightarrow y[n] = e^{j2\pi \hat{f} n} \cdot H(\hat{f})$

if $x[n] = A\cos(2\pi \hat{f} n + \phi) \rightarrow \boxed{h} \rightarrow y[n] = A\cos(2\pi \hat{f} n + \phi + \angle H(\hat{f}))\,|H(\hat{f})|$

how to compute $\overset{h[n]}{\cancel{H(\hat{f})}}$ given $\overset{H(\hat{f})}{\cancel{h[n]}}$.

option A: by inspection

~~recall $H(\hat{f}) =$~~

if $H(\hat{f}) = a_1 e^{-j2\pi \hat{f} k_1} + a_2 e^{-j2\pi \hat{f} k_2} + \ldots$

then $h[n] = a_1 \delta[n-k_1] + a_2 \delta[n-k_2] + \ldots$

option B:

$$h[n] = \int_{-1/2}^{1/2} H(\hat{f}) e^{j2\pi \hat{f} n} \, d\hat{f}$$

option C: use linearity and known h/H pairs!

Filter design

(A) $h[n] = \delta[n] - \delta[n-m]$

$$H(\hat{f}) = \sum_k h[k]\, e^{-j2\pi \hat{f} k}$$

$$= 1 + e^{-j2\pi m \hat{f}}$$

$$= e^{-j\pi m \hat{f}} \left(e^{j\pi m \hat{f}} + e^{-j\pi m \hat{f}} \right)$$

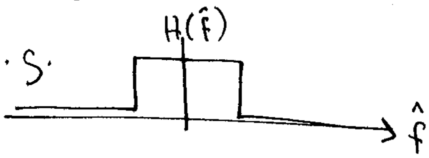
$$= e^{-j\pi m \hat{f}} \cdot 2\cos(\pi m \hat{f})$$

if want to "delete" frequency $\hat{f}_0$, choose m s.t.

$\cos(\pi m \hat{f}_0) = 0$ or $\pi m \hat{f}_0 = \pi/2$ or $m = \dfrac{1}{2\hat{f}_0}$

(B) Start w/ Ideal filter

e.g.

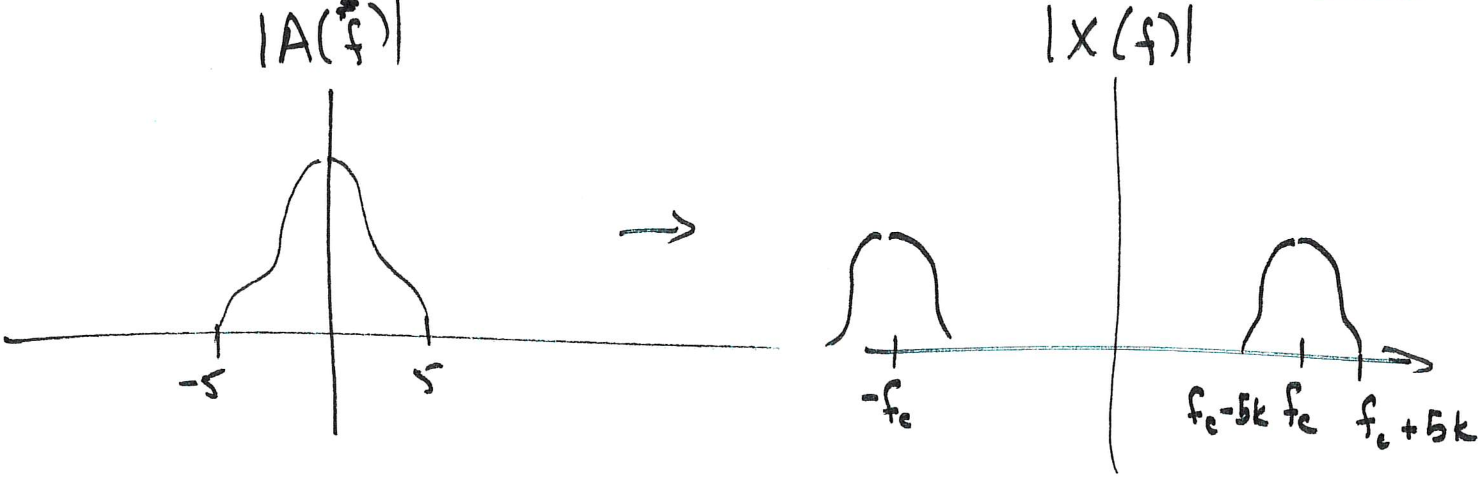


, solve for $h[n]$, truncate.

(C) Parks-McClellan

$$H(\hat{f}) = \sum_k h[k] e^{-j2\pi \hat{f} k}$$

$$\boxed{H(0) = \sum_k \underline{h[k]}}$$



Let $h[n] = \delta[n] + \delta[n-\underline{m}]$

Filter design method A

$$H(\hat{f}) = \sum_k h[k] e^{-j2\pi \hat{f} k}$$

$$= 1 + e^{-j2\pi m \hat{f}}$$

$$= e^{-j\pi m \hat{f}} \left(e^{j\pi m \hat{f}} + e^{-j\pi m \hat{f}} \right)$$

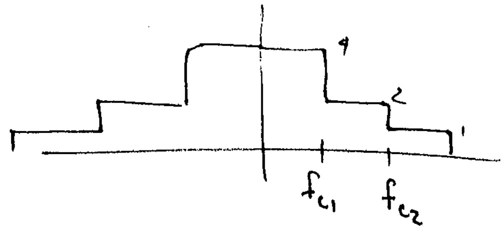
$$= e^{-j\pi m \hat{f}} \cdot 2\cos(\pi m \hat{f})$$

if we want to remove frequency $\hat{f}_0$, then choose m. s.t.

$\cos(\pi m \hat{f}_0) = 0$ or $\pi m \hat{f}_0 = \pi/2$ or $\boxed{m = \frac{1}{2\hat{f}_0}}$

5b)

$$H(\hat{f}) = \begin{cases} 4 & \text{if } |\hat{f}| \leq \hat{f}_{c_1} \\ 2 & \text{if } \hat{f}_{c_1} \leq \hat{f} \leq \hat{f}_{c_2} \\ 1 & \text{else} \end{cases}$$



$$= 2H_3(\hat{f}) + H_2(\hat{f}) + H_1(\hat{f})$$

where $H_1(\hat{f}) = 1 \quad \forall \hat{f}$

$H_2(\hat{f}) = 1$ if $|\hat{f}| \leq f_{c_2}$, 0 o.w.

$H_3(\hat{f}) = \cancel{2}1$ if $|\hat{f}| \leq f_{c_1}$, 0. o.w

$\Rightarrow h[n] = 2h_1[n] + h_2[n] + h_1[n].$

$\hat{f}_{c_1} = .05$

$\hat{f}_{c_2} = .1$

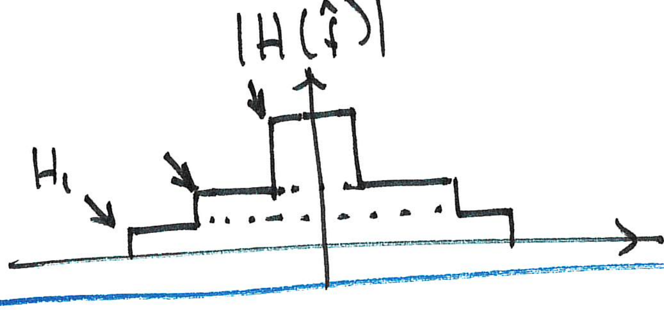
$h_1[n] = \delta[n]$

$$h_2[n] = \begin{cases} \dfrac{\sin(2\pi \hat{f}_{c_2} n)}{\pi n} & \text{if } n \neq 0 \\ 2\hat{f}_{c_2} & \text{if } n = 0 \end{cases}$$

$$h_3[n] = \begin{cases} \dfrac{\sin(2\pi \hat{f}_{c_1} n)}{\pi n} \\ 2\hat{f}_{c_1} \end{cases}$$

$$\Rightarrow h[n] = \begin{cases} \dfrac{\sin(2\pi \hat{f}_{c_1} n)}{\pi n} \cdot 2 + \dfrac{\sin(2\pi \hat{f}_{c_2} n)}{\pi n} & \text{if } n \neq 0 \\ 1 + 2\hat{f}_{c_2} + 4\hat{f}_{c_1} & \text{if } n = 0 \end{cases}$$

$$H(\hat{f}) = \begin{cases} 4 & \text{if } |\hat{f}| < \hat{f}_{c_1} \\ 2 & \text{if } \hat{f}_{c_1} < |\hat{f}| < \hat{f}_{c_2} \\ 1 & \text{otherwise} \end{cases}$$



Recall: if $H(\hat{f}) = \begin{cases} 1 & \text{if } |\hat{f}| < \hat{f}_c \\ 0 & \text{otherwise} \end{cases}$

then $h[n] = \begin{cases} \dfrac{\sin(2\pi \hat{f}_c n)}{\pi n} & \text{if } n \neq 0 \\ 2\hat{f}_c & \text{if } n = 0 \end{cases}$

$$H(\hat{f}) = H_1(\hat{f}) + H_2(\hat{f}) + 2H_3(\hat{f})$$

$H_1(\hat{f}) = 1$ for all $\hat{f}$

$H_2(\hat{f}) = \begin{cases} 1 & \text{if } |\hat{f}| < \hat{f}_{c_2} \\ 0 & \text{otherwise} \end{cases}$

$H_3(\hat{f}) = \begin{cases} 1 & \text{if } |\hat{f}| < \hat{f}_{c_1} \\ 0 & \text{otherwise} \end{cases}$

$$h[n] = h_1[n] + h_2[n] + 2\, h_3[n]$$

$$\boxed{h_1[n] = \delta[n]}$$

$$h_2[n] = \begin{cases} \dfrac{\sin(2\pi \hat{f}_{c_2} n)}{\pi n} & n \neq 0 \\ 2\hat{f}_{c_2} & n = 0 \end{cases}$$

$h_3[n]$ similar

$$\boxed{h[n] = \begin{cases} 1 + 2\hat{f}_{c_2} + 2\hat{f}_{c_1} \cdot 2 & n = 0 \\ \dfrac{\sin(2\pi \hat{f}_{c_2} n)}{\pi n} + \dfrac{2\sin(2\pi \hat{f}_{c_1} n)}{\pi n} & n \neq 0 \end{cases}}$$

7d)

Frequency response of filters:

$$H(\hat{f}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j 2\pi \hat{f} k}$$

if $x[n] = \underline{e^{j2\pi \hat{f} n}} \longrightarrow \boxed{h} \longrightarrow y[n] = \underline{e^{j2\pi \hat{f} n}}\, H(\hat{f})$

$$x[n] = A\cos(2\pi \hat{f} n + \phi) \to \boxed{h} \to y[n] = A\cos(2\pi \hat{f} n + \phi + \angle H(\hat{f})) \cdot |H(\hat{f})|$$

how to compute $h[n]$ given $H(\hat{f})$?

option A: by inspection:

- if $H(\hat{f}) = a_1 e^{-j2\pi \hat{f} k_1} + a_2 e^{-j2\pi \hat{f} k_2}$

$$\Rightarrow h[n] = a_1 \delta[n-k_1] + a_2 \delta[n-k_2]$$

option B:

$$h[n] = \int_{-1/2}^{1/2} H(\hat{f})\, e^{j2\pi \hat{f} n}\, d\hat{f}$$

option C: ~~F~~ use linearity and known h/H pairs.