Lecture 16: Properties of LTI Systems

~~& Frequency Response of FIR filters~~

LTI: Linear, time-invariant

Linear:

if $x_1[n] \rightarrow \boxed{h} \rightarrow y_1[n]$ & $x_2[n] \rightarrow \boxed{h} \rightarrow y_2[n]$

then $a_1 x_1[n] + a_2 x_2[n] \rightarrow \boxed{h} \rightarrow a_1 y_1[n] + a_2 y_2[n]$

Time invariance:

(a) $x[n] \rightarrow \boxed{h} \rightarrow \boxed{\text{delay } n_0} \rightarrow y[n]$

(b) $x[n] \rightarrow \boxed{\text{delay } n_0} \rightarrow \boxed{h} \rightarrow y[n]$

exactly the same for time-invariant systems

> filter operation does not change with time

If $\quad x[n] \longrightarrow \boxed{h} \longrightarrow y[n]$

then $\quad x[n-n_0] \longrightarrow \boxed{h} \longrightarrow y[n-n_0]$

$$\boxed{\text{FIR filters are time-invariant}}$$

If FIR filter has impulse response $h[n]$, then

$$x[n-n_0] * h = \sum_{k=0}^{M} h[k]\, x[n-n_0-k] \qquad \text{let } \ell = n-n_0$$

$$= \sum_{k=0}^{M} h[k]\, x[\ell - k]$$

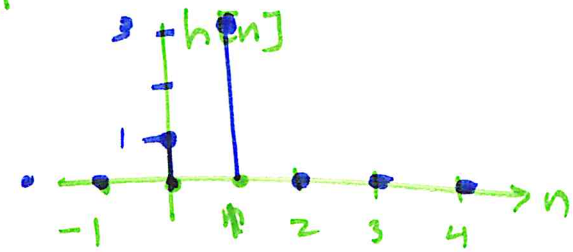
$$= y[\ell] = y[n-n_0]$$

Ex: $y[n] = x[n] + 3x[n-1]$

if $x[n] = \delta[n]$, then $y[n] = \delta[n] + 3\delta[n-1]$

$= h[n]$ = impulse response

$\Rightarrow$ this is an FIR filter



$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

$$\delta[n-1] = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

(a) $x[n] \rightarrow \boxed{h} \rightarrow \boxed{\text{delay } n_0} \rightarrow$

$\Rightarrow x[n] + 3x[n-1] \rightarrow \boxed{\text{delay } n_0}$

$\Rightarrow x[n-n_0] + 3x[n-n_0-1]$

(b) $x[n] \rightarrow \boxed{\text{delay } n_0} \rightarrow h \rightarrow$

$\Rightarrow x[n-n_0] \rightarrow \boxed{h} \rightarrow$

$\Rightarrow x[n-n_0] + 3x[n-n_0-1]$

Ex: $y[n] = \underline{n} \cdot x[n]$

ⓐ $x[n] \longrightarrow \boxed{h} \longrightarrow n\,x[n] \longrightarrow \boxed{\text{delay } n_0} \longrightarrow n\,x[n-n_0]$

ⓑ $x[n] \longrightarrow \boxed{\text{delay } n_0} \longrightarrow x[n-n_0] \longrightarrow \boxed{h} \longrightarrow (n-n_0)\,x[n-n_0]$

$\Rightarrow$ not TI

<u>Causality</u> : output only depends on the past + present input (NOT future) [input "causes" output]
(cannot build non-causal filters)

Ex: $y[n] = x[n] + 3x[n-1]$ — causal

Ex: $y[n] = x[n+1] + 3x[n]$ — non-causal

Properties of LTI systems

Commutative: $y[n] = x[n] * h[n] = h[n] * x[n]$

Associative:

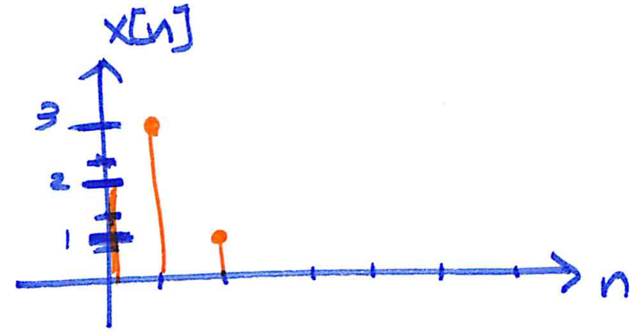
$$y[n] = \left(x[n] * h_1[n]\right) * h_2[n]$$
$$= x[n] * \left(h_1[n] * h_2[n]\right)$$

$$\left.\begin{array}{l}
x[n] \longrightarrow \boxed{h_1} \xrightarrow{x * h_1} \boxed{h_2} \longrightarrow y[n] \\
x[n] \longrightarrow \boxed{h_1 * h_2} \longrightarrow y[n] \\
x[n] \longrightarrow \boxed{h_2} \xrightarrow[x * h_2]{} \boxed{h_1} \longrightarrow y[n]
\end{array}\right\} \text{all equivalent}$$

Using properties:

Any digital signal is a weighted sum of shifted impulses:

$$x[n] = \sum_{m=0}^{N-1} b_m \, \delta[n-m]$$



$$x[n] = \underbrace{2}_{b_0}\,\delta[n] + \underbrace{3}_{b_1}\cdot\delta[n-1] + \underbrace{1}_{b_2}\cdot\delta[n-2]$$

$$x[n] * h[n] = \left(\sum_{m=0}^{N-1} b_m \, \delta[n-m]\right) * h[n]$$

$$= \sum_{m=0}^{N-1} b_m \left(\delta[n-m] * h[n]\right)$$

b/c linear system

$$\delta[n-m] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\,\delta[n-m-k]$$

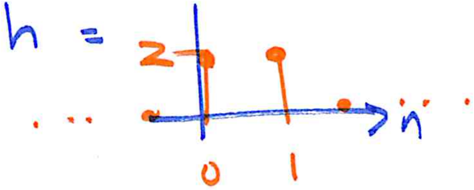
$$= h[n-m]$$

$$= \sum_{m=0}^{N-1} b_m \, h[n-m]$$

~~impu~~

input weighted sum of shift impulses $\longrightarrow$ output = weighted sum of shift impulse responses

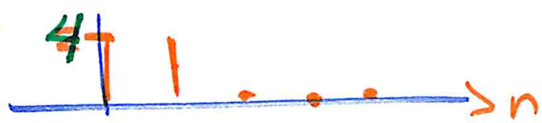
Ex: $h =$



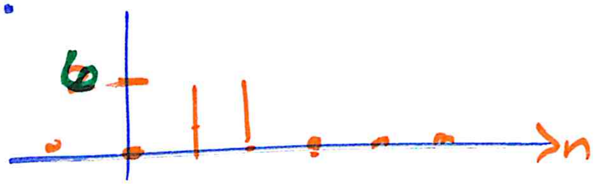
$x[n] * h[n] = \cancel{b_0 h[0] + b_1 h[1]}$

$= b_0 \cdot h[n-0] + b_1 h[n-1] + b_2 h[n-2]$

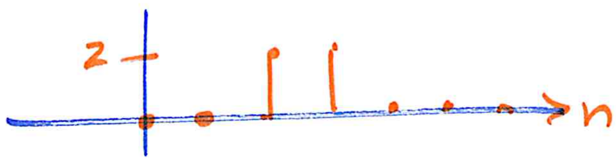
$= \cancel{b_0} \cdot h[n]$



$+ \cancel{b_1} \cdot h[n-1]$



$+ \cancel{b_2} \cdot h[n-2]$



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